Logical form of quantifier phrases: quantifier-sortal-variable

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Abstract: Sentences in mathematical English such as *E* below, expressing propositions about [natural numbers, are routinely translated into sentences such as *F* below of an interpreted first-order language. *E*: Every number *x* is either zero or the successor of some number *y*.
F: $\forall x\ (\ x = o \vee \exists y\ x = sy\ )$. The English quantifier phrases *every number x* and *some number y* correspond to the symbolic quantifier phrases $\forall x$ and $\exists y$. However, there is disagreement amon g mathematica logicians concerning symbolic "location" of the meaning of the sortal [substantive] *number*. Some logicians, the majority, take the meaning o *number* to be elliptically absorbed into the variable so that *x* means, in effect, *number x*. Others take it to be absorbed into the quantifier so tha $\forall$ means, in effect, *every number*. The former, the sortal variable approach, treats the quantifier as a logical symbol but, contrary to tradition, treats the variable as non-logical. The latter, the sortal quantifier approach, treats the variable as logical, but, contrary to tradition, treats the quantifier as non-logical. This paper traces the disagreement to Frege, who did not recognize universes o f discourse, whose individual variables ranged over "the universe", and whose suppressed sortal was *object* (or *individual*), which is naturally taken to be logical. This paper favors a third alternative, the suppressed sortal approach, in effect, tha $\forall x$ and $\exists y$ are elliptical for $\forall Nx$ and $\exists Ny$, where *N* is a sortal, or symbolic [common] noun, a category useful for understanding how number-theoreti c propositions can be fully expressed in first-order. According to this approach, the quantifier and the variable are both logical and the content of the quantifier phrase, $\forall x$ or $\exists y$, is carried by the suppressed non-logical sorta *N*.