DOUGLAS CENZER* AND ANDRE NIES, *Initial segments of the lattice of* Π^0_1 *classes*
Department of Mathematics, University of Florida, Gainesville, FL 32611
Department of Mathematics, University of Chicago, Chicago, IL 60637
e-mail: cenzer@math.ufl.edu

The study of the lattice $\mathcal{E}$ of computably enumerable sets under inclusion has been one of the central tasks of computability theory since the 1960's. We investigate initial segments of the lattice $\mathcal{L}$ of Π^0_1 classes (of sets) under inclusion and we compare this lattice with $\mathcal{E}$. It was recently proved by Nies that the theory of each interval of the lattice $\mathcal{E}$ which is not a boolean algebra interprets true arithmetic (and is therefore certainly undecidable.)

However, we will show that in $\mathcal{L}$ there are initial segments $[\emptyset, P] = \mathcal{L}(P)$ which are not Boolean algebras, but which have a decidable theory. In fact, we will construct for any finite distributive lattice L which satisfies the dual of the usual reduction property a Π^0_1 class P such that L is isomorphic to the lattice $\mathcal{L}(P)$ modulo finite differences.

The construction of the Π^0_1 class corresponding to a given lattice builds on the construction of a *minimal* Π^0_1 class by Cenzer, Downey, Jockusch and Shore in 1993. The simplest minimal Π^0_1 class P has a single limit point together with countably many isolated points. P has the property that every Π^0_1 subclass Q of P is either finite or is cofinite in P – furthermore, Q is the intersection of P with a clopen set. Thus the lattice $[0, P]$ of Π^0_1 subclasses of P is isomorphic to the class of finite/cofinite subsets of ω and is a Boolean algebra. Such a class plays a role in the lattice $\mathcal{L}$ corresponding to the dual of the role played by a maximal c. e. set in the lattice $\mathcal{E}$.

For the simplest new Π^0_1 class P constructed, P has a single, non-computable limit point and $\mathcal{L}(P)^*$ has three elements, corresponding to $\emptyset$, P and a minimal class $P_0 \subset P$. The third type of subclass has no complement in the lattice, which is why the lattice is not a Boolean algebra. On the other hand, the theory of L is shown to be decidable, so that this lattice may not be realized as the class of c. e. subsets of any c. e. set.

A Π^0_1 class P is said to be decidable if it is the set of paths through a computable tree T with no dead ends. We show that if P is decidable and has only finitely many limit points, then $\mathcal{L}(P)^*$ is always a Boolean algebra. We show that if P is a decidable Π^0_1 class and $\mathcal{L}(P)$ is not a Boolean algebra, then the theory of $\mathcal{L}(P)$ interprets the theory of arithmetic and is therefore undecidable.