Epsilon substitution method for theories of jump hierachies

Toshiyasu Arai
Faculty of Integrated Arts and Sciences, Hiroshima University,
Higashi-Hiroshima, 739-8521 Japan
E-mail:arai@mis.hiroshima-u.ac.jp

Let $<$ denote a recursive well ordering of type Γ_0 and α_0 a fixed limit ordinal less than Γ_0. $(H)_{\alpha_0}$ denotes the theory of the absolute jump hierarchy generated by a fixed quantifier-free, arithmetical formula $\mathcal{A}(x,y,z,X)$ and up to $\alpha \leq \alpha_0$ having axioms $\alpha \leq \alpha_0 \rightarrow \{z \in H_\alpha \leftrightarrow \exists x \mathcal{A}(x,\alpha,z,H_{<\alpha})\}$, where $H_{<\alpha} = \sum_{\beta<\alpha} H_\beta$.

We formulate an epsilon substitution method for the ϵ-counter part $(H)_{\alpha_0}\epsilon$ of $(H)_{\alpha_0}$: Replace the existential quantifier in the RHS of the axiom for H by an epsilon term containing the predicate $H_<$. An *epsilon substitution* S is defined to be a function such that each $e \in dom(S)$ is either a canonical ϵ-term with $S(e) \in \omega$ or e is one of formulas $n \in H_\alpha\,(\alpha \leq \alpha_0)$ and $(\beta,n) \in H_{<\alpha}\,(\beta < \alpha \leq \alpha_0)$, and in the latter case $S(e)$ is a boolean value in $\{\bot,\top\}$. We give two termination proofs of the H-process: The first proof is an adaption of [2], [3], i.e., based on a cut-elimination of a specially designed infinitary calculus. The second one is an adaption of [1]. Each termination proof is based on transfinite induction up to an ordinal $\theta\alpha_0\varepsilon_0$, where the normal function $\theta\alpha\beta$ is the α^{th} iterate of the function $\theta 1\beta = \omega^\beta$.

References

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