Name: ____________________

PID: ____________________

Section: ____________________

Question	Points	Score
1	10	
2	5	
3	21	
4	4	
Total:	40	

1. Write your Name, PID, and Section on the front of your Blue Book.
2. Write the Version of your exam on the front of your Blue Book.
3. Hand in the first page of the exam with your Blue book.
4. No calculators or other electronic devices are allowed during this exam.
5. You may use one page of notes, but no books or other assistance during this exam.
6. Read each question carefully, and answer each question completely.
7. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each question on a new page.
8. Show all of your work; no credit will be given for unsupported answers.

1. Let $A = (1, 0, 3)$ and $B = (-3, 2, 1)$.
   (a) (3 points) Normalize $\overrightarrow{AB}$.
   (b) (3 points) Find the midpoint M of the segment AB.
   (c) (4 points) Find equation of the plane passing through M and perpendicular to AB.

2. (5 points) Evaluate the limit or determine that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}.$$

3. Answer the following questions with short justifications:
   (a) (3 points) Find the length of $\frac{-3}{\|\mathbf{v}\|}\mathbf{v}$.
   (b) (4 points) Suppose $\mathbf{v} \times \mathbf{w} = (1, -2, 1)$ and $\mathbf{u} = (1, 1, -1)$. Find the the volume of the parallelepiped spanned by $\mathbf{v}, \mathbf{w}$ and $\mathbf{u}$.
   (c) (4 points) Suppose $\|\mathbf{v} \times \mathbf{w}\| = 3$. Find the area of the parallelogram spanned by $2\mathbf{v} + 3\mathbf{w}$ and $\mathbf{v} + \mathbf{w}$.
   (d) (4 points) Suppose $\|\mathbf{v}\| = 2$, $\|\text{proj}_{\mathbf{v}}\mathbf{w}\| = 5$, and the angle between $\mathbf{v}$ and $\mathbf{w}$ is obtuse. Find $\mathbf{v} \cdot \mathbf{w}$
   (e) (3 points) Find a normal vector of a plane which is parallel to the line $\mathbf{l}(t) = t(1, 2, 3) + (1, 0, 1)$ and perpendicular to the plane $x - y + z = 1$. .
   (f) (3 points) Find a vector parallel to the line of intersection of the planes $x + y + z = 1$ and $-x + y - z = 0$.

4. (4 points) Match the following functions with the contour diagrams (a)-(d).

(1) $f_1(x, y) = x^3 - y$, (2) $f_2(x, y) = xy$, (3) $f_3(x, y) = x^2 - y^2$, (4) $f_4(x, y) = y - \ln x$.

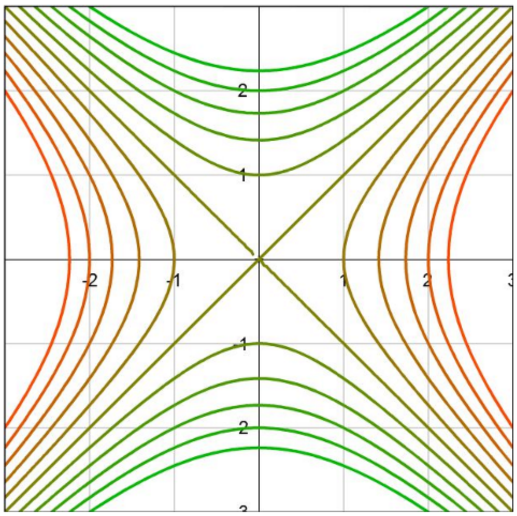


Figure (a)

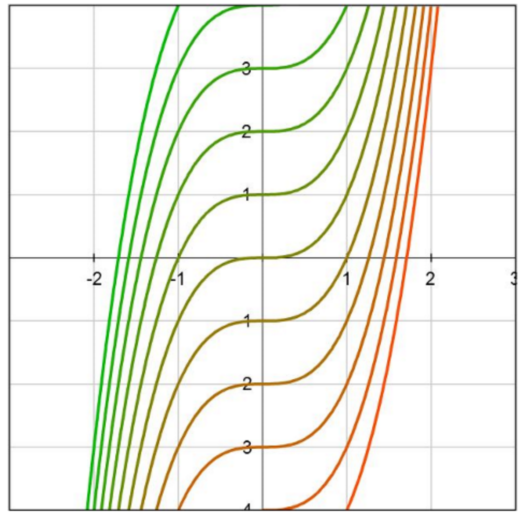


Figure (b)

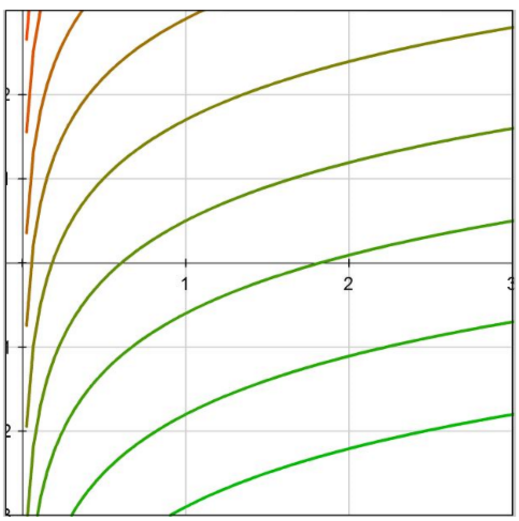


Figure (c)

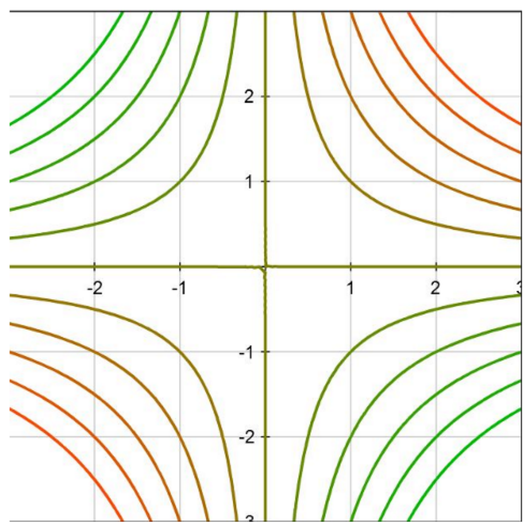


Figure (d)

Good Luck!

1(a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-3,2,1) - (1,0,3) = (-4,2,-2)$

$\|\overrightarrow{AB}\| = \sqrt{(-4)^2 + 2^2 + (-2)^2} = \sqrt{16+4+4} = \sqrt{24}$.

Normalizing $\overrightarrow{AB}$: $\dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \left(\dfrac{-4}{\sqrt{24}}, \dfrac{2}{\sqrt{24}}, \dfrac{-2}{\sqrt{24}}\right)$

$= \left(-\dfrac{\sqrt{24}}{6}, \dfrac{\sqrt{24}}{12}, -\dfrac{\sqrt{24}}{12}\right)$.

Lecture 1, page 2; Lecture 4, page 3.

1(b) $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB} = \left(\frac{-3+1}{2}, \frac{2+0}{2}, \frac{1+3}{2}\right)$

$= (-1,1,2)$. So $M = (-1,1,2)$.

Lecture 3, page 4.

1(c) $-4x + 2y - 2z = (-4)(-1) + 2(1) - 2(2)$

$= 4 + 2 - 4 = 2$

So $-2x + y - z = 1$.

Lecture 8, page 2.

2. Let's approach to $(0,0)$ along the line $y = kx$.

$$\lim_{x \to 0} \frac{x(kx)}{x^2 + (kx)^2} = \lim_{x \to 0} \frac{k\, x^2}{(1+k^2)x^2} = \frac{k}{1+k^2}$$

Since this limit depends on k, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does NOT exist.

Lecture 10, page 5.

3(a) $\dfrac{\vec{v}}{\|\vec{v}\|}$ is a unit vector. So length of $-3\dfrac{\vec{v}}{\|\vec{v}\|}$ is $|-3| = 3$.

Lecture 4, page 3.

3(b) volume of the parallelepiped spanned by $\vec{v}$, $\vec{w}$, and $\vec{u}$

$$= |(\vec{v}\times\vec{w})\cdot\vec{u}| = |(1,-2,1)\cdot(1,1,-1)|$$

$$= |1-2-1| = 2.$$

Lecture 7, page 6.

3(c) Area of the parallelogram spanned by $2\vec{v}+3\vec{w}$ and

$$\vec{v}+\vec{w} = \|(2\vec{v}+3\vec{w})\times(\vec{v}+\vec{w})\|$$

$$= \| \underbrace{\cancel{2\,\vec{v}\times\vec{v}}}_{0} + 2\,\vec{v}\times\vec{w} + 3\,\vec{w}\times\vec{v} + \underbrace{\cancel{3\,\vec{w}\times\vec{w}}}_{0} \|$$

$$= \| 2\,\vec{v}\times\vec{w} - 3\,\vec{v}\times\vec{w} \| = \| -\vec{v}\times\vec{w} \| = 3.$$

Lecture 6, page 4.

3(d) $|\vec{v}\cdot\vec{w}| = \|\mathrm{Proj}_{\vec{v}}\vec{w}\|\,\|\vec{v}\| = (5)(2) = 10$

Since the angle between $\vec{v}$ and $\vec{w}$ is obtuse, $\vec{v}\cdot\vec{w} < 0$.

So $\vec{v}\cdot\vec{w} = -10$.

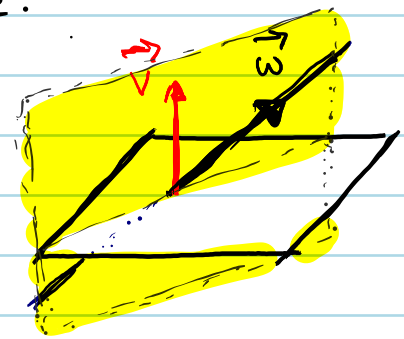
Lecture 5, page 1,2.

3(e) Both normal vector of $x-y+z=1$ and a vector parallel to the line $\overrightarrow{l(t)} = t(1,2,3)+(1,0,1)$ are parallel to the plane that we are looking for. So

$\vec{v} = (1,-1,1)$ and $\vec{w} = (1,2,3)$ are parallel to this plane. Hence $\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = (-3-2, 1-3, 2+1)$

$$= (-5, -2, 3)$$

is a normal vector of this plane.



Section 1.3, Problem 35 (This was a bit tricky, and I treated this as a bonus problem.)

3(f) The line of intersection would be perpendicular to normal vectors of the planes $x+y+z=1$ and $-x+y-z=0$

So it is parallel to their cross product:

$$(1,1,1) \times (-1,1,-1) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix} = (-2-1, -1+1, 1+1)$$

$$= (-3, 0, 2).$$

Most parts of problem 3 are similar to problem 4 in the fifth practice exam.

4. (1) $x^3 - y = 0, 1, -1 \Rightarrow y = x^3$ or $x^3 \pm 1 \Rightarrow$ (b)

(2) $xy = 1$ is a level curve $\Rightarrow$ (d)

(3) $x^2 - y^2 = 0, 1, 2 \Rightarrow$ hyperbolas and crossed lines $\Rightarrow$ (a)

(4) $y - \ln x = 0 \Rightarrow y = \ln x \Rightarrow$ (c)

Lecture 9, page 4.