Print Name: ____________________________ **Section Number:** ______

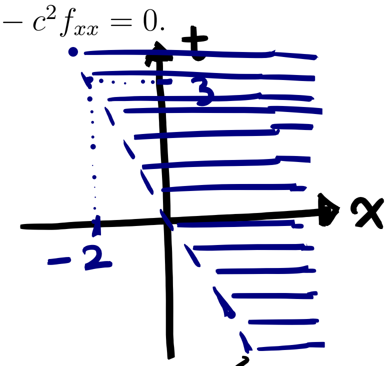
TA Name: ____________________________ **Section Time:** ______

Math 20C.
Midterm Exam 2
May 26, 2006

No calculators or any other devices are allowed on this exam.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Read each question carefully. If any question is not clear, ask for clarification.
Answer each question completely, and show all your work.

1. (a) (5 points) Find and sketch the domain of the function $f(x,t) = \ln(3x+2t)$.
   (b) (5 points) Find all possible constants c such that the function $f(x,t)$ above is solution of the wave equation, $f_{tt} - c^2 f_{xx} = 0$.

(a) $3x + 2t > 0$



line is NOT included

(b) $f(x,t) = \ln(3x+2t)$

$f_x = \dfrac{3}{3x+2t} \Rightarrow f_{xx} = \dfrac{-9}{(3x+2t)^2}.$

$f_t = \dfrac{2}{3x+2t} \Rightarrow f_{tt} = \dfrac{-4}{(3x+2t)^2}.$

$f_{tt} - c^2 f_{xx} = \dfrac{-4 + c^2 9}{(3x+2t)^2} = 0$

if and only if $9c^2 - 4 = 0$

$\Longleftrightarrow \quad c = \pm \dfrac{2}{3}.$

#	Score
1	
2	
3	
4	
Σ	

2. (a) (5 points) Find the direction in which $f(x,y)$ increases the most rapidly, and the directions in which $f(x,y)$ decreases the most rapidly at P_0, and also find the value of the directional derivative of $f(x,y)$ at P_0 along these directions, where

$$f(x,y) = x^3 e^{-2y}, \quad \text{and} \quad P_0 = (1,0).$$

(b) (5 points) Find the directional derivative of $f(x,y)$ above at the point P_0 in the direction given by $\mathbf{v} = \langle 1, -1 \rangle$.

(a) $f(x,y)$ increases the most rapidly in the direction

of $\nabla f(P_0) = \langle 3x^2 e^{-2y}, -2x^3 e^{-2y} \rangle$

$\Rightarrow \nabla f(1,0) = \langle 3, -2 \rangle$.

It decreases the most rapidly in the direction of

$-\nabla f(P_0) = \langle -3, 2 \rangle$.

The directional derivative of f along $\nabla f = \|\nabla f(P_0)\|$

$$= \sqrt{9+4} = \sqrt{13}$$

The directional derivative of f along $-\nabla f = -\|\nabla f(P_0)\|$

$$= -\sqrt{9+4} = -\sqrt{13}$$

(b) $D_u f = \vec{u} \cdot \nabla f$ where $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$.

$$\vec{u} = \frac{\langle 1, -1 \rangle}{\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle$$

$$D_u f(1,0) = \left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle \cdot \langle 3, -2 \rangle$$

$$= \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}.$$

3. (a) (5 points) Find the tangent plane approximation of $f(x, y) = x\cos(\pi y/2) - y^2 e^{-x}$ at the point $(0, 1)$.

(b) (5 points) Use the linear approximation computed above to approximate the value of $f(-0.1, 0.9)$.

(a) $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$$\nabla f = \left\langle \cos\left(\frac{\pi y}{2}\right) + y^2 e^{x} \, , \, -\frac{\pi}{2} x \sin\left(\frac{\pi y}{2}\right) - 2ye^{-x} \right\rangle$$

$\Rightarrow \nabla f(0,1) = \langle 1 \, , -2 \rangle$

and $f(0,1) = -1$

$\Rightarrow \quad z = -1 + (x - 0) - 2(y-1)$

$\Rightarrow \quad z = x - 2y + 1 \, .$

(b) $f(a + \Delta x, b + \Delta y) \approx f(a,b) + f_x(a,b)\Delta x + f_y(a,b)\Delta y$

$\Rightarrow \quad f(-0.1, 0.9) \approx f(0,1) + f_x(0,1)(-0.1) + f_y(0,1)(-0.1)$

$= -1 + (-0.1) + (-2)(-0.1)$

$\Rightarrow \quad f(-0.1, 0.9) \approx -0.9 \, .$

Only the part about local max, local min, or saddle point can be in our exam.

4. (10 points) Find every local and absolute extrema of $f(x,y) = x^2 + 3y^2 + 2y$ on the unit disk $x^2 + y^2 \leq 1$, and indicate which ones are the absolute extrema. In the case of the interior stationary points, decide whether they are local maximum, minimum of saddle points.

- $\nabla f = \langle 2x, 6y+2 \rangle = \langle 0, 0 \rangle$

$\Rightarrow$ $x = 0$ and $y = \frac{-1}{3}$ $\Rightarrow$ $(0, \frac{-1}{3})$ is the only critical pt of f, and it is in the unit disk.

- On the boundary, we use Lagrange multiplier:

$$\begin{cases} \nabla f = c \nabla g & \text{, where } g(x,y) = x^2 + y^2 . \\ g = 1 \end{cases}$$

$$\Rightarrow \quad \langle 2x, 6y+2 \rangle = c \langle 2x, 2y \rangle$$

$$\Rightarrow \begin{cases} x = cx \\ 3y+1 = cy \end{cases} \quad \Rightarrow c = 1 \text{ or } x = 0$$

If $c = 1$, then $3y + 1 = y \Rightarrow y = \frac{-1}{2}$

Since $x^2 + y^2 = 1$, we have $x = \pm \frac{\sqrt{3}}{2}$

If $x = 0$, then $y = \pm 1$ as $x^2 + y^2 = 1$.

So overall on the boundary we get four points that might give us the absolute max or the abs. min.

Pts	$(0, \frac{-1}{3})$	$(\pm \frac{\sqrt{3}}{2}, \frac{-1}{2})$	$(0, 1)$	$(0, -1)$
f	$\frac{-1}{3}$	$\frac{1}{2}$	5	1
	abs. min		abs. max.	

Since $f(0, \frac{-1}{3})$ is an abs. min, it is also a local min.