Math 21C Midterm II, Fall 02, Lindblad.

1. A particle moves with position vector given by $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t^{3/2}\mathbf{k}$.
(a) Find the velocity and speed of the particle at time $t = \pi$.
(b) How far does the particle travel between time $t = 0$ and $t = 10$?

2. Let $F(x, y) = x^3 - x^2 + y^2 - y + 1$
(a) Find the gradient $\nabla F(x, y)$.
(b) Find the directional derivative in the direction of $\langle 1, 2\rangle$ at the point $(2, 3)$.
(c) Let $\mathbf{r}(t) = \langle f(t), g(t)\rangle$ be a curve such that $\mathbf{r}(0) = \langle 1, 0\rangle$ and $\mathbf{r}'(0) = \langle 1, 1\rangle$. Let $h(t) = F(\mathbf{r}(t))$. Find $h'(0)$.

3. Consider the surface given by $z = f(x, y) = \sqrt{x^2 + 3y^2}$.
a) Find the tangent plane to the surface at the point $(1, 1, 2)$.
b) A student was asked to find an approximation for $f(1.1, 1.2)$ but the professor did not allow calculators. The student noticed that $f(1.1, 1.2)$ is approximately $f(1, 1) = \sqrt{1 + 3} = 2$. Use the linear approximation to get a better approximation.

4. Let $f(x, y) = x^3 - x^2 + y^2 - y + 1$. Find the critical points of $f(x, y)$ and determine if they are local max, min or saddle points. Are there any absolute max or min?

5. Use Lagrange multipliers to find the point on the hyperboloid $\{(x, y, z);\ z^2 = x^2 + y^2 + 1,\ z \geq 0\}$ that is closest to the point $(0, 0, -2)$.