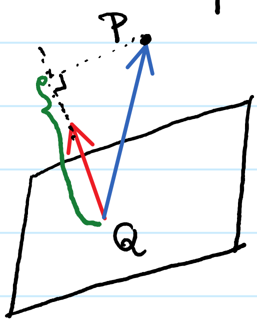
Lecture 9: Distance from a plane

Wednesday, October 12, 2016 8:28 AM

Before we move to the next topic, let me explain the formula on the distance of a point from a plane:



$ax+by+cz=d$.

dist. of P from this plane $= \|\text{Proj}_{\vec{n}_0} \overrightarrow{QP}\|$

$$= \left\| \frac{\overrightarrow{QP}\cdot\vec{n}_0}{\vec{n}_0\cdot\vec{n}_0}\,\vec{n}_0 \right\|$$

$$= \frac{|\overrightarrow{QP}\cdot\vec{n}_0|}{\|\vec{n}_0\|} = \frac{|\overrightarrow{OP}\cdot\vec{n}_0 - \overrightarrow{OQ}\cdot\vec{n}_0|}{\|\vec{n}_0\|}$$

$$= \frac{|ax_0+by_0+cz_0-ax_1-by_1-cz_1|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|ax_0+by_0+cz_0-d|}{\sqrt{a^2+b^2+c^2}}$$

Ex. Find the distance between two parallel planes

$$x+2y+3z=1 \quad \text{and} \quad x+2y+3z=3.$$

Solution. Suppose $P_0=(x_0,y_0,z_0)$ is a point on the first plane. So $x_0+2y_0+3z_0=1$. The distance between these parallel planes = dist. from P_0 to the second plane =

$$\frac{|x_0+2y_0+3z_0-3|}{\sqrt{1^2+2^2+3^2}} = \frac{|1-3|}{\sqrt{14}} = \frac{2\sqrt{14}}{14} = \frac{\sqrt{14}}{7}.$$

Lecture 9: Multivariable functions

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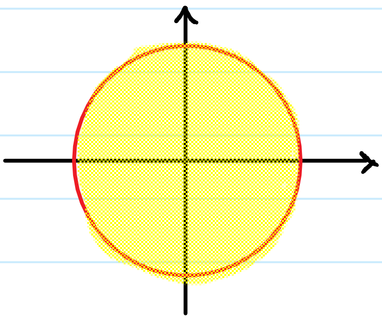
As we discussed before, in life properties of objects depend on many parameters. So we need to work with <u>multivariable functions</u>.

<u>Ex</u> ① $f(x,y) = \ln(9-x^2-y^2)$.

② $g(x,y) = \sqrt{y^2-x}$.

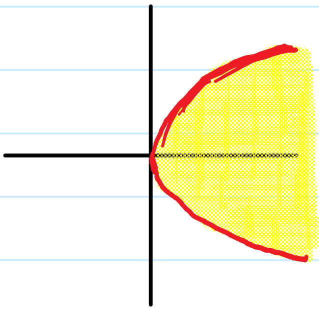
Find domain of the above functions.

① $9-x^2-y^2>0$. So $9>x^2+y^2$



The boundary is NOT included.

② $y^2-x\geq 0$. So $y^2\geq x$.



The boundary is included.

In order to visualize a function and "see" its properties, we sketch its graph (as in the single-variable case.)

Lecture 9: Graph of a two- variable function

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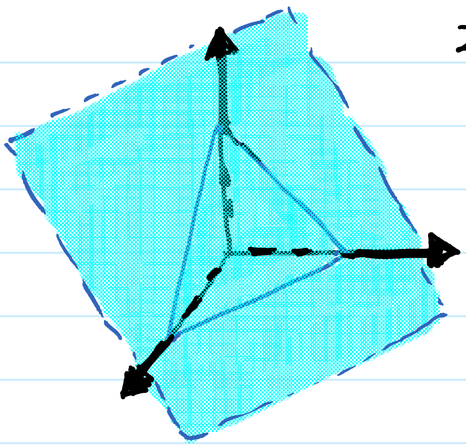
Ex. Sketch graph of $f(x,y) = 1 - x - y$.

Solution. $z = 1 - x - y$. So $x + y + z = 1$. As we know, it is a plane. To sketch it, let's find its intersections with x,y,z-axis:

x-axis $\rightsquigarrow (1,0,0)$

y-axis $\rightsquigarrow (0,1,0)$

z-axis $\rightsquigarrow (0,0,1)$



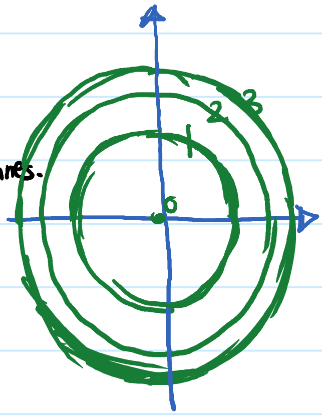
Exp. Sketch the graph of $f(x,y) = x^2 + y^2$.

Solution. Two sketch this, let's use contour map and horizontal traces. I.e. we will look at the intersections of graph $z = x^2 + y^2$ of f and planes parallel to xy, yz, xz-planes.

$x^2 + y^2 = 0$

$x^2 + y^2 = 1$

$x^2 + y^2 = 2$ $\qquad$ $x^2 + y^2 = 3$



Contour map

Lecture 9: Graph of two variable functions

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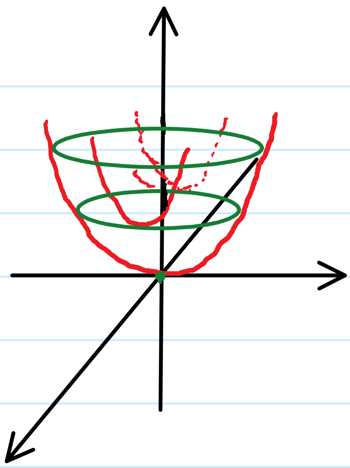
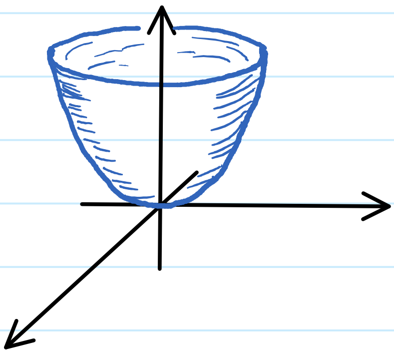
Let's look at intersections of $z=x^2+y^2$ and planes parallel to the yz-plane:



$x=0 \mapsto z=y^2$

$x=\pm 1 \mapsto z=y^2+1$

So $z=x^2+y^2$ (called a paraboloid) looks like

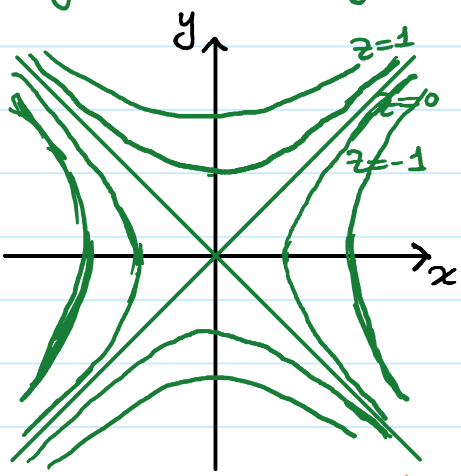


<u>Ex.</u> Sketch graph of $f(x,y)=y^2-x^2$ and its contour map.

<u>Solution.</u> $z=0 \mapsto 0=y^2-x^2$. So $y=\pm x$

$z=1 \mapsto 1=y^2-x^2$

$z=-1 \mapsto -1=y^2-x^2$



At contour curves value of $f(x,y)$ does NOT change.

These maps are used for maps of mountains.

Lecture 9: Graph of a two variable function

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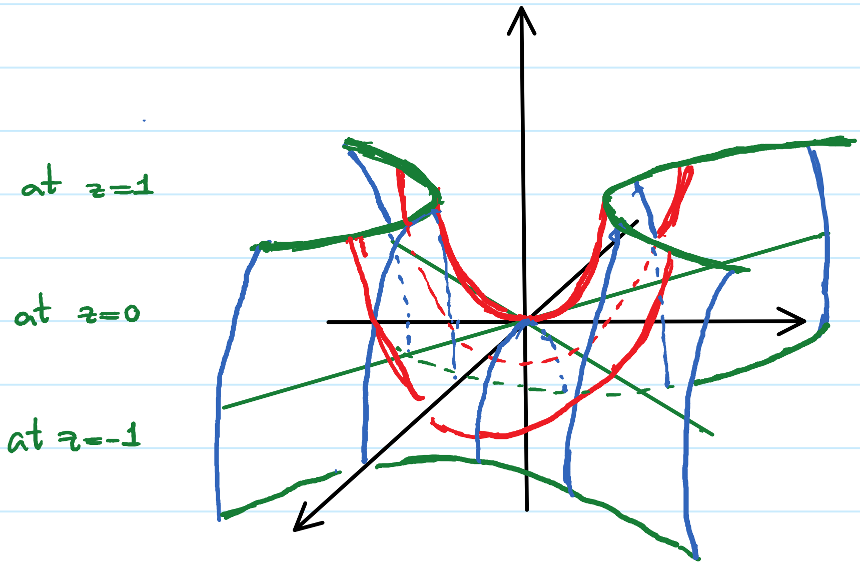

Now intersections with planes parallel to the yz-plane:

$x=0 \longmapsto z=y^2$

$x=\pm 1 \longmapsto z=y^2-1$

Now intersections with planes parallel to the xz-plane:

$y=0 \longmapsto z=-x^2$

$y=\pm 1 \longmapsto z=1-x^2$

So overall $z=y^2-x^2$ looks like a <u>saddle</u>: