Lecture 11: Language of set theory

Monday, October 17, 2016   9:29 AM

In the previous lecture we casually defined a set:

"a well-defined collection of objects." Think about like a box

which contains certain objects.

We mentioned $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$.

Definition. Objects in a set are called its elements or members.

We write $a \in A$ to say a is an element of A or a is in A.

And we write $a \notin A$ to say $\neg(a \in A)$.

Ex. $1/2 \in \mathbb{Q}$, $1/2 \notin \mathbb{Z}$, $i \in \mathbb{C}$, $i \notin \mathbb{R}$.

A set can be given in various ways.

1. List the elements

Ex. $A = \{1, 2\}$.

$1 \in A$, $3 \notin A$, $\{1\} \notin A$.

Ex. $\{\ \}$. This is the empty box

It is called the empty set

It is also denoted by $\emptyset$.

Ex. $B = \{1, \{1, 2\}\}$

$1 \in B$, $2 \notin B$, $\{1\} \notin B$

$\{1, 2\} \in B$

B is a "box" which contains 2 objects: the first object is 1 and the second object is a box which contains 1 and 2.

Lecture 11: Language of set theory

Monday, October 17, 2016 1:49 PM

Ex. $\{\ \} \in \{\{\ \}\}$.

Definition. Two sets A and B are equal if they contain the same collection of members.

"$x \in A \iff x \in B$".

Ex. $\{1,1\} = \{1\}$ Repeating an element does NOT change the set.

Ex. $\{\{1,2\},1\} = \{1, \{1,2,1\}, 1\}$.

Ex. $\{\{\}\} = \{\varnothing, \{\}\}$.

2. Giving the conditions of membership.

Ex. $\cdot \{n \in \mathbb{Z} \mid 2 \mid n\}$ The set of even numbers

$\cdot \{n \in \mathbb{Z} \mid 2 \nmid n\}$ The set of odd numbers

$\cdot \mathbb{H} = \{z \in \mathbb{C} \mid \operatorname{Im}(z) > 0\}$ The upper-half plane.

$\cdot \{x \in \mathbb{R} \mid x^2 - 1 = 0\} = \{1, -1\}$

$\cdot \{x \in \mathbb{R} \mid x^2 - 1 = 0,\ x \geq 2\} = \varnothing$.

3. Constructing the elements of the set

Lecture 11: The language of set theory

Monday, October 17, 2016 2:39 PM

Ex. $\{2k \mid k\in\mathbb{Z}\}$ = the set of even numbers

$$= \{n\in\mathbb{Z} \mid 2 | n\}$$

$$\{2k+1 \mid k\in\mathbb{Z}\} \stackrel{?}{=} \{n\in\mathbb{Z} \mid 2\nmid n\}$$

To prove this we have to show for an integer n

$$\left(\begin{array}{l} n \text{ is of the form} \\ 2k+1 \text{ for some} \\ \text{integer } k \end{array}\right) \iff 2\nmid n,$$

which we have proved before.

Definition. For two sets A and B, we say A is a subset of B and write $A\subseteq B$ if any element of A is in B.

"$x\in A \Rightarrow x\in B$".

Ex. . $\{1\}\subseteq\{1,2\}$

. $\{\{1\}\}\not\subseteq\{1,2\}$ since $\{1\}\in\{\{1\}\} \wedge \{1\}\notin\{1,2\}$.

To show $A\not\subseteq B$ it is enough to find x such that

$$x\in A \wedge x\notin B.$$

. $\{\}\subseteq\{1\}$

In fact $\emptyset\subseteq A$ for any set A.

Ex. Give two sets A and B such that $A\in B \wedge A\subseteq B$.

Solution. $A=\{1\}$ and $B=\{1,\{1\}\}$ are one such example.
$A=\emptyset$ and $B=\{\emptyset\}$ is another.