

## Lecture 4: Linear Diophantine equation

Friday, September 30, 2016 9:02 AM

In the previous lecture we proved

Lemma. For any integers  $a$  and  $b$ ,

$$(a|b \wedge b \neq 0) \Rightarrow |a| \leq |b|.$$

Let's see some of its applications:

Q. Does the equation  $14m - 49n = 1$  have integer solutions? (This type of equations are called Diophantine equations.)

Solution. No! Suppose to the contrary that there are integers  $m$  and  $n$  such that

$$14m - 49n = 1.$$

Then the left hand side  $14m - 49n = 7(2m - 7n)$  is a multiple of 7 as  $2m - 7n$  is an integer.

Hence  $7 | 1$ . By the above lemma we get

$$|7| \leq |1|,$$

which is a contradiction. ■

# Lecture 4: Diophantine

Friday, September 30, 2016 9:23 AM

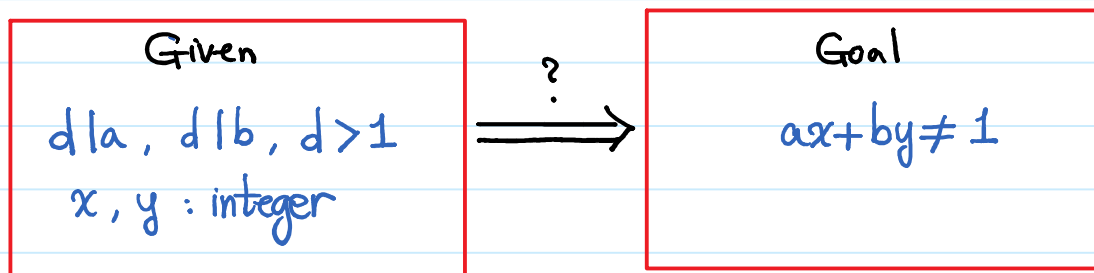
The same argument implies.

Lemma. Suppose  $a$  and  $b$  are two integers.

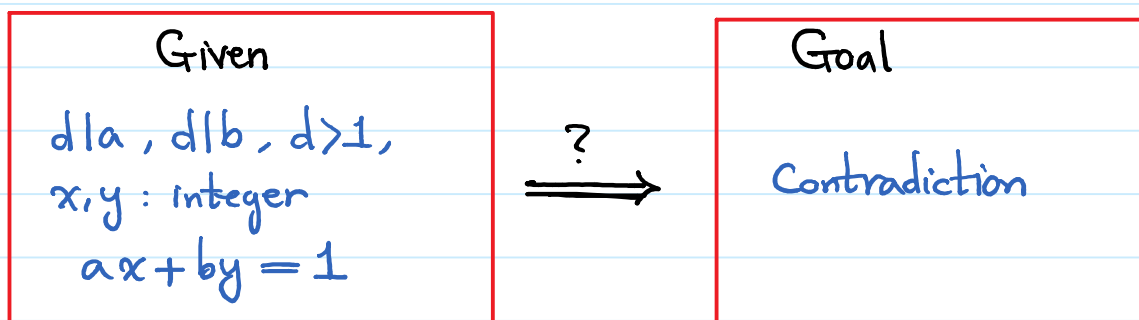
If  $a$  and  $b$  have a common divisor  $d$  greater than 1,

then the equation  $ax+by=1$  has no integer solutions.

Draft / Proof.



Proof by contradiction



$$\begin{aligned} d|a &\Rightarrow \text{for some integer } a' & \Rightarrow ax+by &= da'x+db'y \\ & a = da' & & = d(a'x+b'y) \\ d|b &\Rightarrow \text{for some integer } b' & & \\ & b = db' & & \\ & & \Rightarrow d | ax+by = 1 & \\ & & \xrightarrow{\text{by lemma}} |d| \leq 1, & \\ & & \text{which is a contradiction. } \blacksquare & \end{aligned}$$

## Lecture 4: Biconditional proposition, odd and even

Friday, September 30, 2016 9:38 AM

In fact, the converse of this lemma is also correct, but it is harder to prove. We will do it later in this course.

Converse of  $P \Rightarrow Q$  is  $Q \Rightarrow P$ . In general  $P \Rightarrow Q$

might be true and at the same time  $Q \Rightarrow P$  be false.

Biconditional Proposition  $P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$ .

.  $P$  if and only if  $Q$ .

.  $P$  is necessary and sufficient for  $Q$ .

.  $P \Leftrightarrow Q$  is true exactly when  $P$  and  $Q$  have the same truth value.

$P$	$Q$	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$
<u>T</u>	<u>T</u>	T	T	<u>T</u>
T	F	F	T	F
F	T	T	F	F
<u>F</u>	<u>F</u>	T	T	<u>T</u>

Definition. Let  $n$  be an integer. We say  $n$  is even if  $2 \mid n$ .

We say  $n$  is odd if  $n$  is NOT even.

Important remark. Since the above conditional proposition is defining a phrase, it gets promoted to a biconditional proposition.