Lecture 2: Russell's example.

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Ex. This is a false sentence.

. It is NOT a proposition.

Suppose to the contrary that it is a proposition.

(This method of proof is called proof by contradiction.)

Then there are two possibilities:

(This method of arguing is called case-by-case proof.)

Case 1. It is a true proposition. So the claim of this proposition is supposed to be true, which says it is false. That is a contradiction.

Case 2. It is a false proposition. So the claim of this proposition is supposed to be false, which implies that it is true. That is a contradiction.

The only reason that we are getting a contradiction is because we assumed that the above sentence is a proposition. Hence this is NOT a valid assumption, i.e. the above sentence is NOT a proposition.

Lecture 2: Propositional forms and truth tables

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A <u>propositional form</u> is a legitimate expression involving logical variables, connectives $\wedge, \vee, \Rightarrow, \Leftrightarrow, \neg$, and (,).

Here are the truth table of conjunction $\wedge$, disjunction $\vee$, and implication $\Rightarrow$.

P and Q, $P \wedge Q$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

It is "and" so both should hold for P and Q to hold

all the possibilities of the truth values of P, Q.

. If there are 3 varibles, the number of rows is 8.

For 4 variables, we get 16 rows. For n variables, we get 2^n rows.

P or Q, $P \vee Q$

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

For this to fail, both P and Q should fail

This is slightly different from the way "or" is used in a daily language. There we sometimes assume P and Q do not hold at the same time.

P implies Q, $P \Rightarrow Q$

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication fails only when hypothesis (P) holds and conclusion (Q) fails.

In particular $P \Rightarrow Q$ is true if P is false.

Lecture 2: Equivalence of propositional forms

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Ex. $(1=2) \Rightarrow$ Sun is a moon.

is a true proposition as the hypothesis is false.

Of course I am NOT claiming that the conclusion "sun is a moon" is correct. We are only saying that the implication is true.

Ex. Write the truth table of $\neg(P \vee Q)$ and $(\neg P) \wedge (\neg Q)$.

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$(\neg P) \wedge (\neg Q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Definition Two propositional forms are called equivalent if they have the same truth tables.

Ex. $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$

Similarly $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$

} de Morgan's law.

Convention. $\top$: a true proposition.

$\perp$: a false proposition. (Contradiction).

Lecture 2: Proof by contradiction

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Ex. (Proof by contradiction) To prove P it is equivalent to assume (not P) and reach to a contradiction.

$$P \equiv (\neg P) \Rightarrow \perp .$$

P	$\neg P$	$\perp$	$(\neg P) \Rightarrow \perp$
T	F	F	T
F	T	F	F

(as the hypoth. is false)

(as the hyp. holds and the conclusion fails.)