

Problem set 4

Thursday, October 20, 2016 11:29 PM

1. (a) List elements of $\{A \in \mathcal{P}(\{1, 2, 3, 4\}) \mid |A| \text{ is even}\}$.

(b) List elements of $\{A \in \mathcal{P}(\{1, 2, 3, 4\}) \mid |A| \text{ is odd}\}$.

2. Determine the truth-value of the following propositions:

(a) $|\{\{1\}, \{x \in \mathbb{R} \mid x > 0, (x^2 - 1)^2 = 0\}\}| = 1$.

(b) $\{\emptyset\} \subseteq \{1, \{\emptyset\}\}$.

(c) $|\{1, \mathbb{R}, \{x \in \mathbb{R} \mid x^2 \geq 0\}\}| = 2$.

3. Let X be a set.

(a) Prove that, for any $A \subseteq X$, we have $A \Delta \emptyset = A$

(a) Prove that, for any $A \subseteq X$, we have $A \Delta A = \emptyset$.

(b) Prove that, for any $A, B, C \subseteq X$, we have

$$A \Delta B = A \Delta C \Rightarrow B = C$$

(Hint. You are allowed to use $(E \Delta F) \Delta G = E \Delta (F \Delta G)$.)

4. Prove, for any positive integer n and $A \subseteq \{1, 2, \dots, n\}$,

we have $|A| \text{ is even} \iff |A \Delta \{1\}| \text{ is odd}$.

5. Let X be a set. Prove that for any $A, B, C \subseteq X$ we have:

(a) $A \subseteq B \iff A \cap B = A$.

(b) $((A \cap B = A \cap C) \wedge (A \cup B = A \cup C)) \Rightarrow B = C$.