Problem set 2

Tuesday, October 4, 2016 11:06 PM

1. Prove that, for any integer n, $2 \mid n(n+1)$.

2. Suppose p is prime, i.e. $p \mid ab \Rightarrow p \mid a \vee p \mid b$ for integers a, b, and $p > 1$. Prove that for integers a, b

$$p = ab \Rightarrow (p = \pm a \vee p = \pm b).$$

3. For integers d, a, b, prove that

$$(d \mid a \wedge a \mid b) \Rightarrow d \mid b.$$

4. Prove that for integers d, m, n, r, and s,

$$\left.\begin{array}{l} d \mid m \\ d \mid n \end{array}\right\} \Rightarrow d \mid rm + sn.$$

5. Is it true or false?

For any integer a, b, $6 \mid ab \Rightarrow (6 \mid a \vee 6 \mid b)$.

6. Prove that, for any integer $n > 1$,

$$\left.\begin{array}{l} n \text{ has a} \\ \text{divisor } d \text{ such that} \\ 1 < d < n \end{array}\right\} \Rightarrow \left\{\begin{array}{l} n \text{ has a divisor } d' \\ \text{such that} \\ 1 < d' \leq \sqrt{n}. \end{array}\right.$$

7. Prove for any positive real numbers x, y,

$$\sqrt{xy} \geq \frac{2}{\frac{1}{x} + \frac{1}{y}}.$$