1. Prove that $P(\{1,2,\cdots,n\})$ has 2^n elements.

(Hint. Use induction on n. And follow an argument presented in class i.e. split the subsets into two groups:

those that do not contain $n+1$, and those that do contain $n+1$.)

2. List the elements of $X = \{A \subseteq \{1,2,3\} \mid |A| \text{ is even}\}$

and the elements of $Y = \{A \subseteq \{1,2,3\} \mid |A| \text{ is odd}\}$.

3. Find the truth-value of the following statements; justify your answer:

(a) There are No sets A and B such that

$$A \in B \wedge A \subseteq B.$$

(b) $\{\emptyset\} \subseteq \{1, 2, \{\emptyset\}\}$.

4. Prove that, for any two sets A and B,

$$A \subseteq B \Leftrightarrow A = A \cap B$$

(Hint. Use a similar argument as in the lecture.)

5. (a) Prove that $A \triangle A = \emptyset$ and $A \triangle \emptyset = A$

(b) Prove that $A \triangle B = A \triangle C \Rightarrow B = C$

(Hint. You are allowed to use

$$X \triangle (Y \triangle Z) = (X \triangle Y) \triangle Z$$

without proof.)

6. Use quantifiers to write down that $A \subseteq \mathbb{R}$ does NOT have a minimum.