Problem set 2

Monday, October 12, 2015 4:54 PM

1. (a) Prove that for any integer n one and exactly one of the numbers n and $n+1$ is even.

(b) Prove that, for any integer n,

$n(n+1)$ is even.

(Hint. For (a) use the result that we proved in class:

For any integer m, m is odd if and only if $m = 2k+1$ for some integer k.

For (b), use part (a).)

2. Prove that $201x - 9y = 2$ has no integer solutions.

3. Prove that for any positive real numbers x, y, z

$$\sqrt{\frac{x^2+y^2+z^2}{3}} \geq \frac{x+y+z}{3}.$$

(You are allowed to use whatever is proved in class.)

4. Determine if the following statements are true or not. Justify your answer.

(a) For any integers m and n,

$$6 \mid mn \implies 6 \mid m \vee 6 \mid n.$$

(b) For any integers m and n,

$$6 \mid m \vee 6 \mid n \Rightarrow 6 \mid mn.$$

(c) For any integers m and n,

$$3 \mid mn \Rightarrow 3 \mid m \vee 3 \mid n$$

(For part (c) you are allowed to use the following:

For any integer n, $3 \nmid n \iff$ for some integer k, $n = 3k \pm 1$.)

5. Let d be an integer more than 1, and a_1, a_2, b_1, and b_2 are integers. Suppose $d \mid a_1 - a_2$ and $d \mid b_1 - b_2$.

Prove that $d \mid (a_1 + b_1) - (a_2 + b_2)$.

and $d \mid a_1 b_1 - a_2 b_2$.

(Hint. For the second part use

$$a_1 b_1 - a_2 b_2 = (a_1 - a_2) b_1 + a_2 (b_1 - b_2).)$$