Wednesday, October 21, 2015 11:07 AM

In the previous lecture we defined $x \in A$, $A \subseteq B$.

Ex. For any set A and a (well-defined) object x,

$$x \in A \iff \{x\} \subseteq A.$$

Ex./Def. The power set $\mathcal{P}(X)$ of a set X is

$$\mathcal{P}(X) = \{ A \mid A \subseteq X \}.$$

Ex. List the elements of $\mathcal{P}(\{1,2\})$.

Solution. $\mathcal{P}(\{1,2\}) = \{ \emptyset, \{1\}, \{2\}, \{1,2\} \}$.

Ex. List the elements of $\mathcal{P}(\{1,2,3\})$

Solution. Suppose $A \subseteq \{1,2,3\}$. Either $3 \in A$ or $3 \notin A$.

	Contains 3	Does NOT contain 3
The rest is a subset of $\{1,2\}$	$\{3\}$	$\{\ \}$
	$\{1,3\}$	$\{1\}$
	$\{2,3\}$	$\{2\}$
	$\{1,2,3\}$	$\{1,2\}$.

$\square$

Remark List the elements: 1, 2, 3

For each element you have two choices
put it or NOT put it

1	2	3		
Y	Y	Y	$\rightsquigarrow$	$\{1,2,3\}$
Y	Y	N	$\rightsquigarrow$	$\{1,2\}$
Y	N	Y	$\rightsquigarrow$	$\{1,3\}$
Y		N	$\rightsquigarrow$	$\{1\}$

you have two choices
put it or NOT put it
in the subset

Y	N	Y	$\rightsquigarrow \{1,3\}$
Y	N	N	$\rightsquigarrow \{1\}$
N	Y	Y	$\rightsquigarrow \{2,3\}$
N	Y	N	$\rightsquigarrow \{2\}$
N	N	Y	$\rightsquigarrow \{3\}$
N	N	N	$\rightsquigarrow \{\,\}$

The same point of view gives us that the number of elements of $\mathcal{P}(X)$ is $2^{|X|}$ if X has $|X|$ many elements.

Operations on sets

$A \cap B$: the intersection of A and B.

$A \cup B$: the union of A and B.

$A \setminus B$: the difference of A and B.

A^c : the complement of $A \in \mathcal{P}(X)$.

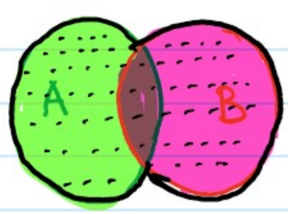
and my favorite operation:

$A \triangle B$: the symmetric difference.

Visualize these operations before giving the formal definitions:

Venn diagram

the union of A and B

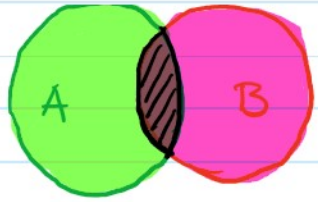


$A \cup B$

the intersection of

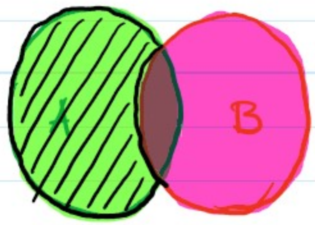
$A \cap B$

the intersection of A and B



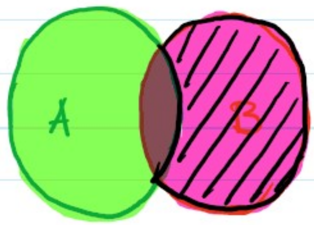
$A \cap B$

the difference of A and B



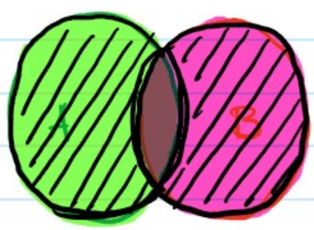
$A \setminus B$

the difference of B and A



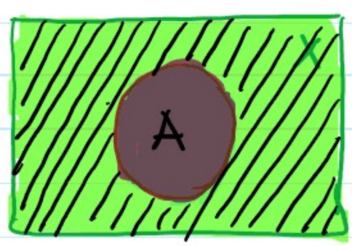
$B \setminus A$

the symmetric difference of A and B



$A \triangle B$

the complement of A (in X)



A^c

<u>Definitions</u>.

$$x \in A \cap B \iff x \in A \wedge x \in B$$

$$x \in A \cup B \iff x \in A \vee x \in B$$

$$x \in A \setminus B \iff x \in A \wedge x \notin B$$

$$x \in A \triangle B \iff x \in A \cup B \wedge x \notin A \cap B$$

( ) (

Ex. Let $A = \{1,2,3\}$ and $B = \{3,4\}$. Then

$$A \cap B = \{3\}$$

$$A \cup B = \{1,2,3,4\}$$

$$A \setminus B = \{1,2\}$$

$$B \setminus A = \{4\}$$

$$A \triangle B = \{1,2,4\}.$$

Proposition. $A \triangle B = (A \cup B) \setminus (A \cap B)$

$$= (A \setminus B) \cup (B \setminus A).$$

In the next lecture we use a truth-table to prove this proposition.