Truth table, equivalent propositional forms.

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Recall what a proposition is.

Exp. For any real number x, we have $x^2 > 0$.

It is a false proposition. Here this claim is supposed to hold for every real number. So it is enough to show a single example where it fails. This is called counter-example. This appoach only works when someone claim a property is supposed to hold for everything (in certain domain).

(For instance, is my claim true that all the students here are male?)

Ex. This is a false sentence.

It is NOT a proposition. To show this we essentially use proof by contradiction.

Suppose it is a proposition. Then it is either true or false.

Case 1. Suppose it is true.

Then it implies that it is false. Since it cannot be both true or false, we get a contradiction. This implies that Case 1 cannot happen.

Case 2. Suppose it is false.

Then it implies that it is true. So again it is a contradi., which implies the second case cannot happen, either.

So we should question our assumption that it is a proposition. Hence it is NOT a proposition. ■

Remark. We have used two techniques of proving statements: case-by-case and prove by contradiction. We will see more mathematical examples of such methods.

Propositional forms and truth table.

Towards the end of the previous class we introduced

$P \vee Q$: disjunction and $P \wedge Q$: conjunction

These kind of formulas are called propositional form: (legitimate) expression involving logical variables, connectives, (,).

We also talked about the truth table of a propositional form.

Ex. Write the truth table of the following propositional forms.

$(\neg P) \vee Q$, $\neg(P \wedge Q)$, $(\neg P) \vee (\neg Q)$.

Solution. We break the propositional forms into smaller propositional forms in order to deal with a single connective at a time.

forms in order to deal with a single connective at a time.

so we need to have $P, Q, \neg P, \neg Q, P \wedge Q$ in the truth table (in addition to the above propositional forms.)

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$(\neg P) \vee Q$	$\neg(P \wedge Q)$	$(\neg P) \vee (\neg Q)$
T	T	F	F	T	T	F	F
T	F	F	T	F	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T
I	II	III	IV	V			
		based on I	based on II	based on I, II	based on III, II	based on V	based on III, IV

We observe that $\neg(P \wedge Q)$ and $(\neg P) \vee (\neg Q)$ have the same truth value in all the cases. In another word, one is true if and only if the other one is true.

<u>Def.</u> We say two propositions are equivalent if one implies the other one and vice versa.

. Two propositional forms are called equivalent if they have the same truth value in all the cases.

Ex. $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$

$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$

(de Morgan's law).

As we have seen in these two lectures an extremely important logical connective is implication.

Conditional propositions also known as implications.

$P \Rightarrow Q$ we say P implies Q.

If P, then Q.

P is sufficient for Q.

Q is necessary for P.

Here is its truth table

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

How should we think about its truth table?

If I claim $P \Rightarrow Q$, then the only way for you to show me that my claim is false is by showing an instance that P is true, but Q is false.

For instance, if I claim that "If a student is enrolled in this class, then he is male.", then you say "no, it is NOT true, you say "no, it is NOT true, because my friend has enrolled in this class and she is a female." So you provide a case where the assumption is true, but the conclusion is false.