Mathematical statements and logical connectives

Friday, September 25, 2015 11:16 AM

1. In this course you are supposed to learn
   - How to listen to a proof and understand it.
   - How to read a proof and understand it.
   - How to produce a proof and communicate your thoughts.

2. The key to success is doing lots of exercises.

3. I will convey the new concepts using different techniques, eg. playing! Present examples, and prove important statements.

4. I expect that you read the text before each lecture, go over my notes as I post them on the course webpage.

   math.ucsd.edu/~asalehig/math109-15-f.html

   go to discussion sessions, come to my office hours with your questions and suggestions,
   do all the homework assignments,
   come to all the lectures (even if you are sleepy.)

5. Please let me know if
   - you cannot hear me.
   - you cannot read something on the blackboard.

Mathematical Language.

Proposition : a sentence that is either true or false.

Warning. In math books or articles, you see

theorem, lemma, corollary, definition, proposition, etc.

is supposed to be true.

Ex. ⓐ $1+1$

ⓑ $1+1=2$

ⓒ $m=1$

ⓓ If m is integer and $0<m<2$, then $m=1$.

ⓔ If x, y, z are positive integers and n is a positive integer more than 3, then

$$x^n+y^n \neq z^n.$$

ⓕ $x^2 \geq 0$

ⓖ For any real number x, $x^2 \geq 0$.

Examples ⓒ and ⓕ are called predicates and m and x are called free variables.

Let's look at the above propositions again. Pay attension to connectives, negation, quantifiers, implication.

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During lectures we saw the difference between

$1+1$ not being a proposition and $m=1$ not being a proposition.

The second one has a free variable and can be made into a proposition by adding a quantifier, which essentially tells us where and in what capacity should we look for the free variable. We will discuss this later.

Logical connectives.

We discussed a few examples from calculus to see the need of connectives. Then we discussed the truth value of (P and Q) and (P or Q).

We expected two things:

- if (P and Q) is true, then
  (P is true) and (Q is true).
- if (P or Q) is false, then
  (P is false) and (Q is false).

We summarized these in the following tables:

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P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F