Name: ____________________

PID: ____________________

Section: ____________________

Question	Points	Score
1	12	
2	8	
3	10	
Total:	30	

1. Write your Name, PID, and Section on the front page of your exam.
2. No smart phone or other electronic devices are allowed during this exam.
3. If you have to leave the lecture hall during the exam for any reason, you have to leave your exam and smart phone or other electronic devices on the table in front of the lecture hall with the proctor.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in the exam sheet.
6. If you are using a result proved in lectures, you have to clearly write down the statement of this result.
7. Show all of your work; no credit will be given for unsupported answers.

1. (a) (4 points) Find all the primes p such that $x - 2$ is a factor of

$$x^5 - x^4 + 2x^3 - x^2 + x - 1$$

in $\mathbb{Z}_p[x]$.

(b) (4 points) True or false. A polynomial of degree 2 over $\mathbb{Z}_6$ has at most 2 zeros. (Justify your answer)

(c) (4 points) Write down a version of Gauss's lemma. (Define all the technical terms that you are using.)

2. In each part, prove that the given polynomial does not have a zero in the given field.

   (a) (4 points) $x^7 - x + 2$ in $\mathbb{Z}_7$.

   (b) (4 points) $x^7 - 7x^5 + 14x^3 - 8x + 2$ in $\mathbb{Q}$.

3. Let $\alpha \in \mathbb{C}$ be a zero of $f_0(x) = x^5 - 12x^4 + 24x^3 - 18x^2 + 6x - 6$ and

$$\mathbb{Q}[\alpha] = \{\sum_{i=0}^{n} a_i \alpha^i |\ a_i \in \mathbb{Q}, n \in \mathbb{Z}^+\}.$$

(a) (4 points) Prove that $f_0(x)$ is irreducible in $\mathbb{Q}[x]$.

(b) (3 points) Suppose that $m_\alpha(x)$ is the minimal polynomial of α over $\mathbb{Q}$. Prove that $m_\alpha(x) = f_0(x)$.

(c) (3 points) Prove that $\mathbb{Q}[x]/f_0(x)\mathbb{Q}[x] \simeq \mathbb{Q}[\alpha]$.

Good Luck!