Name: ____________________

PID: ____________________

Section: ____________________

Question	Points	Score
1	12	
2	6	
3	8	
4	14	
Total:	40	

1. Write your Name, PID, and Section on the front page of your exam.
2. No smart phone or other electronic devices are allowed during this exam.
3. If you have to leave the lecture hall during the exam for any reason, you have to leave your exam and smart phone or other electronic devices on the table in front of the lecture hall with the proctor.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in the exam sheet.
6. Show all of your work; no credit will be given for unsupported answers.

1. Make the following computations in the given ring.

   (a) (4 points) $(11, 3)(12, 23)$ in $\mathbb{Z}_{22} \times \mathbb{Z}_{46}$.

   (b) (4 points) $(1 - 2x)^{-1}$ in $\mathbb{Z}_{16}[x]$.

   (c) (4 points) $(3^{-1})(2)$ in $\mathbb{Z}_{13}$.

2. Find the characteristic of the following rings. Justify your answer.
   (a) (3 points) $\mathbb{Z}_6 \times \mathbb{Z}_{14} \times \mathbb{Z}_{21}$.

   (b) (3 points) $2\mathbb{Z}_{10}$.

3. (8 points) We are told that $\mathbb{Q}[\sqrt{3}] = \{a + b\sqrt{3} | \, a, b \in \mathbb{Q}\}$ is a subring of $\mathbb{R}$. Show that it is a field. (You do not have to prove that $\sqrt{3}$ is an irrational number.)

4. Answer the following questions; justify your answer only when it is asked:
   (a) (2 points) True or False: Any integral domain is a field. (Justify your answer)

   (b) (2 points) True or False: A finite integral domain is a field.

   (c) (2 points) True or False: there is an integral domain D such that

   $$1_D + 1_D + 1_D \neq 0 \text{ and } \underbrace{1_D + \cdots + 1_D}_{9 \text{ times}} = 0.$$

   (Justify your answer)

   (d) (2 points) True or False: $\mathbb{Z}_{13}[i]$ is a field. (Justify your answer)

   (e) (2 points) True or False: A finite field is of the form $\mathbb{Z}_p$ where p is prime.

   (f) (4 points) Find $|\mathbb{Z}_{100}^{\times}|$. (Justify your answer)

Good Luck!