First problem set

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1. Suppose $R_1, \dots, R_n$ are rings. Prove that $R_1, \dots, R_n$ are unital if and only if $R_1 \times \cdots \times R_n$ is unital.

2. Suppose R is a unital ring. An element x of R is called a <u>unit</u> if it has a multiplicative inverse; that means $\exists\, x' \in R$ such that $x x' = x' x = 1_R$.

   Let $U(R)$ be the set of all the units of R.

   (a) Prove that $U(R)$ is closed under multiplication.

   (b) Prove that $(U(R), \cdot)$ is a group.

   (c) Suppose R_i's are unital rings. Prove that

   $$U(R_1 \times \cdots \times R_n) = U(R_1) \times \cdots \times U(R_n).$$

   (d) Find $U(\mathbb{Z} \times \mathbb{Q})$.

3. Show that $\{a + b\sqrt{3} \mid a, b \in \mathbb{Z}\}$ is ring.

4. As in problem 3 one can show $F = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}$ is a ring. Show that $U(F) = F \setminus \{0\}$; that means any non-zero element is a unit.