THE SECOND MIDTERM.

MATH 103B

The exam will be considered out 50.

1. (10 points) Is $(3/7)x^4 - (2/7)x^2 + (9/35)x + (3/5)$ irreducible over $\mathbb{Q}$?

2.

(1) (5 points) Find all the degree 2 irreducible monic polynomials in $(\mathbb{Z}/3\mathbb{Z})[x]$.
(2) (5 points) Prove that $x^3 - x + 1$ is irreducible over $\mathbb{Z}/3\mathbb{Z}$.
(3) (10 points) Find a field of order 27. Prove your claim.

3. (15 points) Define the content of a polynomial and prove that product of two primitive polynomials is a primitive polynomial.

4. Let R and S be two unital commutative rings and $f : R \to S$ be a surjective ring homomorphism.

(1) (5 points) If I is an ideal of S, then $f^{-1}(I) := \{x \in R|\ f(x) \in I\}$ is an ideal of R.
(2) (5 points) If I is a prime ideal of S, then $f^{-1}(I)$ is a prime ideal of S.
(3) (10 points) If J is an ideal of R, $\ker(f) \subseteq J$ and $f(J) = S$, then $J = R$.
(4) (5 points) If I is a maximal ideal of S, then $f^{-1}(I)$ is a maximal ideal of R.

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