THE SECOND MIDTERM.

MATH 103B

The exam will be considered out 50.

1. (10 points) Is $x^4 + x + 1$ irreducible over $\mathbb{Z}$?

2. (10 points) Construct a field of order 25.

3.

   (1) (3 points) Define the content of a polynomial in $\mathbb{Z}[x]$.
   (2) (12 points) Assume that product of two primitive polynomials is primitive. Prove that $c(fg) = c(f)c(g)$, where $f, g \in \mathbb{Z}[x]$ and $c(f), c(g)$ are the contents of f and g, respectively.

4. For a commutative unital ring R, let $J(R)$ be the intersection of all the maximal ideals of R.

   (1) (10 points) Find all the maximal ideals of $\mathbb{Z}$ and compute $J(\mathbb{Z})$.
   (2) (10 points) Find $J(\mathbb{Z}/8\mathbb{Z})$.
   (3) (10 points) Let R be a commutative unital ring, F be a field and $f : R \to F$ be a surjective ring homomorphism. Then $J(R) \subseteq \ker(f)$.
   (4) (5 points) Let R be a unital commutative ring. Assume that any ideal I of R is contained in a maximal ideal. Prove that for any $x \in J(R)$, $1 + x$ is a unit. (Hint: Consider $\langle 1 + x \rangle$.)

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