LECTURE 23.

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1. Recall

$\mathbb{Z}[\sqrt{10}]$ is not a UFD.

Theorem 1. *$\mathbb{Z}[i]$ is a ED.*

Proof. It has been outlined in one of the this week's problems. □

2. Vector spaces

Definition 2. A set V is called a vector space over F if

(1) V is an abelian group. (It is usually considered an additive group.)
(2) There is a "scalar multiplication", i.e. for any $c \in F$ and $v \in V$, we can "multiply" v by c and write it as cv. The scalar multiplication has the following properties:
 (a) $c(c'v) = (cc')v$.
 (b) $c(v + v') = cv + cv'$.
 (c) $(c + c')v = cv + c'v$.
 (d) $1v = v$.

Members of V are called vectors and members of F are called scalars.

Example 3. (1) *$\mathbb{R}^n$ is a vector space over $\mathbb{R}$.*
(2) *(A bit more general) For any field F, F^n is a vector space over F.*
(3) *(Even more general) Let R be a unital ring and let F be a subring of R with the same unity. Then R is a vector space over F.*

Example 4. (1) *$\mathrm{M}_n(\mathbb{Z}/p\mathbb{Z})$ is a vector space over $\mathbb{Z}/p\mathbb{Z}$.*
(2) *$\mathbb{Q}[x]$ is a vector space over $\mathbb{Q}$.*
(3) *$\mathbb{Q}[i]$ is a vector space over $\mathbb{Z}$.*
(4) *$\mathbb{Z}[i]$ is not a vector space over $\mathbb{Q}$.*

As always, one has to study "substructures" and "homomorphisms": subspaces and linear maps.

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