LECTURE 1.

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In the first lecture, we mainly introduced new definitions and mentioned several examples. In this note, I will just highlight what we did.

Warning: Reading these notes is not enough by any means. You have to also read your book.

(1) $(G, \cdot)$ is a semigroup if
  (a) G is closed under multiplication, i.e.
$$\forall a, b \in G, a \cdot b \in G.$$
  (b) $\cdot$ is associative, i.e.
$$\forall a, b, c \in G, (a \cdot b) \cdot c = a \cdot (b \cdot c).$$
(2) $(G, \cdot)$ is a monoid if
  (a) $(G, \cdot)$ is a semigroup.
  (b) G has an identity, i.e.
$$\exists e \in G, \forall g \in G, eg = ge = g.$$
(3) $(R, +, \cdot)$ is a ring if
  (a) $(R, +)$ is a commutative group.
  (b) $(R, \cdot)$ is a semigroup.
  (c) (Distribution property) $\forall a, b, c \in R, a(b + c) = ab + ac$ & $(b + c)a = ba + ca$.
(4) A ring R is called unital if $(R, \cdot)$ is a monoid.
(5) A ring R is called commutative if $\forall a, b \in R, ab = ba$.
(6) Let R be a unital ring. a is called a left-inverse of b if $ab = 1$. Similarly one can defined a right-inverse.
(7) Let R be a unital ring. $a \in R$ is called invertible or a unit if it has both a left-inverse and a right-inverse. The set of all the units in R is denoted by $U(R)$.

Here are some of the examples that we discussed:

(1) $(\mathbb{Z}, \times)$ is a monoid but it is not a group.
(2) $(\mathbb{Z} \setminus \{1\}, \times)$ is not a semigroup.
(3) $(2\mathbb{Z}, \times)$ is a semigroup but it is not a monoid.
(4) $(\mathbb{Z}, +, \times)$ is a ring.
(5) $(\mathbb{N}, +, \times)$ is not a ring.
(6) The set $\mathrm{M}_n(\mathbb{R})$ of n-by-n real matrices form a non-commutative ring.
(7) $U(\mathrm{M}_n(\mathbb{R})) = \mathrm{GL}_n(\mathbb{R})$.
(8) $(\mathrm{GL}_n(\mathbb{R}), +, \cdot)$ is not a ring!
(9) If $x \in \mathrm{M}_n(\mathbb{R})$ has a left-inverse, then it is invertible.

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