PRACTICE MIDTERM I, MATH 103B, WINTER 2012.

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1. Either provide an example to support your answer or prove your claim: (5 points each)

   (1) What is a unit in a unital ring?
   (2) Give a ring R and a non-principal ideal I.
   (3) Is there a non-commutative ring of order 4?
   (4) Let R be a ring and assume that it has a subring isomorphic to $\mathbb{Q}$. Does R have a unity?

2. Let $S = \{\begin{bmatrix} a & b \\ 2b & a \end{bmatrix} \mid a, b \in \mathbb{Q}\} \subseteq \mathrm{M}_2(\mathbb{Q})$.

   (1) (10 points) Prove that S is a commutative unital subring of $\mathrm{M}_2(\mathbb{Q})$.
   (2) (5 points) Prove that S is a field.
   (3) (10 points) Prove that $f : S \to \mathbb{Q}[\sqrt{2}]$ given by
   $$f(\begin{bmatrix} a & b \\ 2b & a \end{bmatrix}) = a + \sqrt{2}b$$
   is an isomorphism.
   (4) (5 points) Is $R = \{\begin{bmatrix} a & b \\ 2b & a \end{bmatrix} \mid a, b \in \mathbb{R}\}$ a field? Explain your answer.
   (5) (Bonus) Prove that R is isomorphic to $\mathbb{R} \oplus \mathbb{R}$.

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