

Mathematics 204A Homework 4

Due: Friday 27 October 2023

1. (Neukirch, section 3, exercise 5). Let R be the quotient ring $\mathcal{O}/\mathfrak{a}$ of a Dedekind domain by a nonzero ideal $\mathfrak{a}$. Prove that every ideal of R is principal. (See the hint in the text.)
2. (Neukirch, section 3, exercise 6). Prove that every ideal of a Dedekind domain can be generated by two elements. (Use the previous problem.)
3. (Neukirch, section 3, Exercise 8) Let $\mathfrak{m}$ be a nonzero integral ideal of the Dedekind domain $\mathfrak{O}$. Show that in every ideal class of Cl_K , there exists an integral ideal prime to $\mathfrak{m}$. (Hint: You may find it helpful to use the Chinese remainder theorem.)
4. (Murty-Esmonde, Exercise 4.5.4) Suppose α has minimal polynomial $f(x) = x^n + ax + b$. Show that $disc(1, \alpha, \dots, \alpha^{n-1}) = (-1)^{n(n-1)/2}(n^n b^{n-1} + a^n(1-n)^{n-1})$.
5. (Keith Conrad, Expository papers, blurb on “Totally ramified primes and Eisenstein polynomials”). Read Conrad’s proof of the following fact: Let $K = \mathbf{Q}(\alpha)$ where $\alpha \in \mathfrak{O}_K$ is a root of an Eisenstein polynomial at p . Then $p \nmid [\mathfrak{O}_K : \mathbf{Z}[\alpha]]$. (You don’t need to turn in anything for this problem.)