

Mathematics 204A Homework 1

Due: Friday 6 October 2023

1. (part of Neukirch, Ch. 1, section 1, Exercise 7). Show that the ring $\mathbf{Z}[\sqrt{2}]$ is Euclidean. Hint: Consider the function $\psi(a + b\sqrt{2}) = |a^2 - 2b^2|$.
2. (part of Neukirch, Ch. 1, section 1, Exercise 7). Show that the units of the ring $\mathbf{Z}[\sqrt{2}]$ are $\pm(1 + \sqrt{2})^n$, $n \in \mathbf{Z}$, and determine the prime elements of $\mathbf{Z}[\sqrt{2}]$.
3. (part of Neukirch, Ch. 1, section 2, Exercise 4) Let D be a squarefree integer with $D \neq 0, 1$. Consider $K = \mathbf{Q}(\sqrt{D})$. Show that the ring of integers of K is $\mathbf{Z}[\sqrt{D}]$ if D is 2 or 3 mod 4, and is $\mathbf{Z}[(1 + \sqrt{D})/2]$ if D is 1 mod 4.
4. Do the problems in section 2 of the worksheet on binary quadratic forms and Gauss composition. (In later weeks, I will assign some homework questions from sections 3 and 4 of this worksheet.) You do not need to turn anything in for this problem.