Mathematics 204A Homework 4
Due: Friday 29 October 2021

1. (Neukirch, section 5, exercise 2) Show that the convex, centrally symmetric set $X = \{(z_\tau) \in K_{\mathbf{R}} : \sum_\tau |z_\tau| < t\}$ has volume $vol(X) = 2^r \pi^s \frac{t^n}{n!}$. You do not need to turn in anything for this question, but just read the proof in III.2.15.

2. (Neukirch, section 5, exercise 3) Show that in every ideal $\mathfrak{a} \neq 0$ of $\mathcal{O}_K$ there exists an $a \neq 0$ such that $|N_{K/\mathbf{Q}}(a)| \leq M(\mathcal{O}_K : \mathfrak{a})$ where $M = \frac{n!}{n^n} \left(\frac{4}{\pi}\right)^s \sqrt{|d_K|}$. (See the hint in the text.)

3. (Neukirch, section 6, exercise 3) Show that in every ideal class of an algebraic number field K of degree n, there exists an integral ideal $\mathfrak{a}$ such that $N(\mathfrak{a}) \leq M$, where M is the constant of the previous exercise.

4. (Neukirch, section 6, exercise 4) Show that the absolute value of the discriminant $|d_K|$ is > 1 for every algebraic number field $K \neq \mathbf{Q}$.

5. (Neukirch, section 6, exercise 7) Show that, for every number field K, there exists a finite extension L such that every ideal of K becomes a principal ideal.