

Mathematics 105 Homework 1

Due: Friday 14 January 2022

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *An Introduction to Number Theory*, by Harold M. Stark. Please feel free to reach out to me or the TA if you have any questions.

1. Chapter 1, Miscellaneous Exercise 1.
2. The following is a fact, that we'll prove later in the course: If p is prime, and a is an integer with $(a, p) = 1$, then p divides $a^{p-1} - 1$. Use a computer to check this for $p = 11$ and $1 \leq a \leq 10$. Namely, for each integer a with $1 \leq a \leq 10$, evaluate $\frac{1}{11}(a^{10} - 1)$.
3. Let p and q be two consecutive odd members of the sequence of primes $2, 3, 5, 7, 11, \dots$. Show that $p + q$ factors into at least three (not necessarily distinct) primes. (This is a slight rewording of Chapter 1, Miscellaneous Exercise 8.)
4. Chapter 2, section 1, exercise 3.
5. Chapter 2, section 1, exercise 8.
6. Chapter 2, section 1, exercise 9.
7. Chapter 2, section 1, exercise 12.