Mathematics 105 Exam 2 Review Problems

Instructions: The following problems are meant to help you in your studying for Exam 2. Anything covered in lecture up to and including the lecture of Friday February 18th is fair game for Exam 1. Note that inclusion of a topic on this sheet does not guarantee that a similar problem will appear on the exam, nor does exclusion of a topic from this sheet imply that that topic will not be on the exam.

1. The integer 3 is a primitive root modulo the prime 17 (take this as given.) What is the order of 3^6 modulo 17?

2. What are the last three digits of 7^{402}?

3. Suppose p a prime that is congruent to 3 modulo 4. Prove that p is not the sum of two rational squares.

4. Find all solutions x to $x^2 - 3x + 2 \equiv 0 \pmod{41}$.

5. Find an integer x that satisfies $x \equiv 1 \pmod 8$ and $x \equiv 3 \pmod{17}$.

6. Solve the congruence equations: $3x + 2y \equiv 1 \pmod{11}$ and $2x + 3y \equiv 3 \pmod{11}$.

7. Show that for infinitely many integers n, $29 | n^3 + 2n + 26$.

8. Find all solutions in integers (positive, negative, or 0) to $x^2 - 3xy + 2y^2 = 9$. **Hint**: Factor the left-hand side.

9. Find all rational solutions to $2x^2 + 3y^2 = 2$.

10. Show that the equation $2x^2 + 2y^2 + z^2 = 2w^2$ has infinitely many integer solutions with $gcd(w, x, y, z) = 1$. **Hint**: Substitute $w = x + y$.

11. Show that the equation $x^2 - 2y^2 = 5$ has no integral solutions.

12. Find all solutions to the equation $z^5 = x^2 - y^2$ with x, y, z relatively prime positive integers, and z odd.