Mathematics 100C Homework 5
Due: Friday May 17 2024

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers, if any, refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TA if you have any questions.

1. (Chapter 16, Problem 2.2(a)) Prove that the discriminant of a real cubic is nonnegative if and only if the cubic has three real roots.

2. (Chapter 16, Problem 3.2) Determine the degrees of the splitting field of the following polynomials over $\mathbf{Q}$:

   (a) $x^3 - 2$

   (b) $x^4 - 1$

   (c) $x^4 + 1$

3. (Chapter 16, Problem 4.1(b)) We will prove in class that if F has characteristic 0, and K is a splitting field of a polynomial over F, then the size of $\mathrm{Aut}(K/F)$ is equal to the degree of the extension $[K : F]$. Using this result if you'd like, determine all automorphisms of the field K, where K is the splitting field of the polynomial $f(x) = (x^2 - 2x - 1)(x^2 - 2x - 7)$ over the rational numbers $F = \mathbf{Q}$.