

Mathematics 100C Homework 9

Due: Wednesday June 7 2023

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. Chapter 10, exercise 1.1
2. Chapter 10, exercise 2.2. (Note that, in Artin's notation, D_n is the dihedral group of order $2n$.)
3. Chapter 10, exercise 3.5
4. The spectral theorem says that if T is an operator that commutes with its adjoint T^* , then T is diagonalizable. Look up a proof of this theorem, for example, Theorem 8.6.6 in the textbook. You do not need to turn in anything for this question.
5. Another theorem from linear algebra asserts that if linear operators A, B on a vector space V commute, and each are diagonalizable, then V has a basis of common eigenvectors for A, B . Look up a proof of this theorem, or give a proof yourself. You do not need to turn in anything for this question.
6. Suppose G is a finite abelian group, and $\rho : G \rightarrow \mathrm{GL}_n(\mathbf{C})$ is an irreducible representation of G . Prove that $n = 1$. **Hint:** Use the previous two problems. (We will later give a less direct proof of this result.)