

Mathematics 100C Homework 6

Due: Wednesday May 17 2023

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. Chapter 16, exercise 7.5.
2. Chapter 16, exercise 7.6. (Assume F is characteristic 0).
3. Chapter 16, exercise 7.7.
4. Chapter 16, exercise 7.11. For part (a), please only (provably) determine the roots of f in $\mathbf{C}$, you do not need to find $f(x) \in \mathbf{Q}[x]$ exactly. Also, I would do (b) first, then do (a) using your result from (b).
5. Suppose F is a field, and K, L are extensions of F contained in some (large) field E . Let $L = K \cdot F'$ be the compositum of K and F' in E .
 - (a) Recall that if R is a subring of E containing F' , all of whose elements are algebraic over F' , then R is a field. (You do not need to write anything for this part; just recall that we proved this in 100B.)
 - (b) Suppose K/F is finite. Prove that L is equal to the set of finite sums $x_1y_1 + \cdots + x_ky_k$ with $x_j \in K$ and $y_j \in F'$. **Hint:** Let R be the set of these finite sums. Now use the previous part.
 - (c) Suppose $b_1, \dots, b_m$ is a spanning set of K/F . Prove that $b_1, \dots, b_m$ span L over F' .
 - (d) Now suppose F'/F is finite, and $c_1, \dots, c_n$ is a spanning set of F'/F . Deduce that the elements b_ic_j , with $1 \leq i \leq m$ and $1 \leq j \leq n$ span L over F .