

Mathematics 100C Homework 1

Due: Wednesday April 12 2023

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. Suppose G is a finite abelian group. Let $n = \max_{g \in G} \text{ord}(g)$. Suppose $y \in G$ and $m = \text{ord}(y)$. Prove that m divides n .
2. Chapter 15, exercise 6.1.
3. Chapter 15, exercise 6.2
4. Let F be a finite field of characteristic p . Denote by $\sigma : F \rightarrow F$ the map $\sigma(a) = a^p$. We already know that σ is a ring homomorphism. Prove that σ is an automorphism of F , i.e., that σ is bijective.
5. Chapter 15, exercise 7.10. **Hint:** You might find it useful to use the previous problem.