

Mathematics 100B Homework 8

Due: Wednesday March 8 2023

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. Let α be a complex number. Prove that the kernel of the substitution map $\mathbf{Z}[x] \rightarrow \mathbf{C}$ given by $x \mapsto \alpha$ is a principal ideal of $\mathbf{Z}[x]$.
2. Chapter 12, Exercise 4.3
3. Chapter 12, Exercise 4.5
4. Chapter 12, Exercise 5.6
5. Let F be a field, V, W two vector spaces over F , and $\varphi : V \rightarrow W$ a surjective linear map. Suppose V is finite-dimensional. Prove that W is finite-dimensional, and $\dim(W) \leq \dim(V)$.