Mathematics 100B Homework 5
Due: Wednesday February 15 2023

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. Chapter 11, exercise 7.1

2. Let R be an integral domain and F its field of fractions. Let $a \in R$ be nonzero. Let ϕ be the map $R[t] \to F$ given by $t \mapsto a^{-1}$, and let $\overline{\phi} : R[t]/(at-1) \to F$ be the induced map. Prove that $\overline{\phi}$ is injective. **Hint**: Suppose $f(t) = a_n t^n + \cdots + a_0$ is in the kernel of ϕ. Let $f_1(u) \in R[u]$ be the polynomial $a_0 u^n + \cdots + a_n$. Observe that $f_1(a) = 0$, so that $f_1(u) = (u-a)h_1(u)$ for some polynomial $h_1 \in R[u]$ of degree $n-1$. Let F_1 be the field of fractions of $R[t]$. We can evaluate $u = t^{-1}$ in the field F_1 to obtain $f_1(t^{-1}) = (t^{-1} - a)h_1(t^{-1})$. Multiplying both sides of this equality by t^n, one gets $f(t) = (1-at)h(t)$ for some polynomial $h \in R[t]$, as needed.

3. Let F be a field, and $R = F[[x]]$ the power series ring in the variable x over the field F. Let S be the ring obtained by inverting x, i.e., $S = R[t]/(xt-1)$. Prove that S is isomorphic to the fraction field of R. **Hint**: Apply the previous problem.

4. Chapter 11, exercise 8.2

5. Chapter 11, exercise 8.3