

## Mathematics 100B Homework 3

### Due: Wednesday February 1 2023

**Instructions:** Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. Chapter 11, Exercise 3.12
2. Chapter 11, Exercise 3.13
3. Suppose  $\varphi : R \rightarrow S$  is a ring homomorphism, and  $I \subseteq R$  is an ideal with  $I \subseteq \ker(\varphi)$ . Let  $\pi : R \rightarrow \overline{R} = R/I$  denote the quotient map. You know from group theory that there exists a unique map of abelian groups  $f : R/I \rightarrow S$  such that  $\varphi = f \circ \pi$ . Prove that  $f$  is a ring homomorphism.
4. Identify explicitly the ring  $\mathbf{Z}[x]/(6, 2x - 1)$ .
5. Set  $R = \mathbf{Z}/2\mathbf{Z}$ . Prove that the rings  $\mathbf{Z}[x]/(x^2 - 3, 2x + 4)$  and  $R[t]/(t^2)$  are isomorphic. Construct an explicit isomorphism between them. How many elements does this ring have?