

Mathematics 100B Homework 1

Due: Wednesday January 18 2023

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. Chapter 11, Exercise 1.2. **Hint:** Here is one way to proceed. Define the so-called Chebyshev polynomials by the recurrence relation $T_0(x) = 1$, $T_1(x) = x$ and $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$. Using expressions for $\cos(n\theta + \theta)$ and $\cos(n\theta - \theta)$, prove that $T_n(\cos(\theta)) = \cos(n\theta)$. By specializing θ to $2\pi/n$, deduce the problem statement. **Note:** Later in the term, we will give a much simpler way of solving this problem.
2. Chapter 11, Exercise 1.5. Note that a subset T of \mathbf{R} is said to be discrete if, given any element $t \in T$, there exists $\epsilon > 0$ so that if $t' \in T$ and $t' \neq t$, then $|t - t'| \geq \epsilon$.
3. Chapter 11, Exercise 1.6 (a)
4. Chapter 11, Exercise 1.8. For part (c), feel free to use Corollary 2.3.6 in the text.
5. Chapter 11, Exercise 2.1.