

Mathematics 100A Homework 9

Due: Wednesday 2 December 2020

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Please feel free to reach out to me or the TA if you have any questions.

1. Chapter 6, exercise 10.2
2. Denote by S_n the symmetric group of permutations of $\{1, 2, \dots, n\}$. Define a map $S_n \times \mathbf{Z}^n \rightarrow \mathbf{Z}^n$ as

$$(p, (a_1, a_2, \dots, a_n)) \mapsto (a_{p^{-1}(1)}, a_{p^{-1}(2)}, \dots, a_{p^{-1}(n)}).$$

Here $p \in S_n$ and $a_1, a_2, \dots, a_n \in \mathbf{Z}$. Prove that this map defines an action of S_n on $\mathbf{Z}^n$ by group homomorphisms.

3. Suppose A is an abelian group, and a group G acts on A by group homomorphisms. Denote $A^G = \{a \in A : g * a = a \forall g \in G\}$.
 - (a) Prove that A^G is contained in the center of the semidirect product $A \rtimes_{\phi} G$, where $\phi : G \rightarrow \text{Aut}(A)$ is the homomorphism derived from the action of G on A .
 - (b) Let $Z(G)$ denote the center of G , and denote by H the subgroup of $Z(G)$ defined as

$$H = \{z \in Z(G) : z * a = a \forall a \in A\}.$$

Prove that the center of $A \rtimes_{\phi} G$ is the direct product $A^G \times H$.