

Mathematics 100A Homework 5

Due: Wednesday 4 November 2020

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Please feel free to reach out to me or the TA if you have any questions.

1. Chapter 2, exercise 9.3
2. Chapter 2, exercise 9.5
3. Chapter 2, exercise 9.7
4. This question has two parts.
 - (a) Suppose p is a permutation in S_n , and that $p = (123 \cdots k)$ for some $k \leq n$. That is, $p(1) = 2, p(2) = 3, \dots, p(k) = 1$ and $p(j) = j$ if $k + 1 \leq j \leq n$. Prove that $\text{sgn}(p) = (-1)^{k-1}$. **Hint:** Induct on k .
 - (b) Suppose $n_1, n_2, \dots, n_k$ are distinct integers between 1 and n inclusive. Let q be the permutation of S_n with cycle decomposition $q = (n_1 n_2 \cdots n_k)$. Prove that q is conjugate to p . Deduce that $\text{sgn}(q) = (-1)^{k-1}$. **Hint:** Let g be any permutation in S_n with $g(1) = n_1, g(2) = n_2, \dots, g(k) = n_k$. Consider gpg^{-1} .
5. Chapter 2, exercise 10.2
6. Chapter 2, exercise 12.4