

Mathematics 100A Homework 4

Due: Wednesday 28 October 2020

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Please feel free to reach out to me or the TA if you have any questions.

1. Chapter 2, exercise 6.11
2. Chapter 2, exercise 8.3
3. Chapter 2, exercise 8.6
4. Chapter 2, exercise 8.7
5. Chapter 2, exercise 8.10
6. This question has three parts.
 - (a) For which residue classes x modulo 15 does one have $x^2 \equiv 1 \pmod{15}$?
 - (b) Suppose p is a prime number. For which residue classes x modulo p does one have $x^2 \equiv 1 \pmod{p}$?
 - (c) Suppose p is an odd prime number and a is an integer relatively prime to p . Set $x = a^{\frac{p-1}{2}}$. Prove that $x \equiv \pm 1 \pmod{p}$. **Hint:** Prove that $x^2 \equiv 1 \pmod{p}$ using a result from Lecture 10, and apply Part (b).