

Mathematics 100A Homework 3

Due: Wednesday 21 October 2020

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Please feel free to reach out to me or the TA if you have any questions.

1. In class, we defined a linear transformation $L(p)$ of $\mathbf{R}^n$ associated to a permutation p in the symmetric group S_n . Recall that if e_j is the j^{th} standard basis vector of $\mathbf{R}^n$, then $L(p)e_j = e_{p(j)}$. Prove that $\det(L(p)) = \pm 1$.

Here is one way that you can proceed: check that $L(p)$ preserves the standard inner product on $\mathbf{R}^n$. Deduce that ${}^tL(p)L(p) = 1$ and then take determinants to conclude that $\det(L(p))^2 = 1$.

2. Prove that if $n \geq 3$, then the center of the symmetric group S_n is the trivial subgroup $\{1\}$.

Here is one way that you can proceed: Suppose p is in the center. One wants to prove that $p(j) = j$ for all j . To see that $p(1) = 1$, consider the fact that p commutes with the permutation $g = (234 \cdots n)$. Argue similarly for other $j \neq 1$.

3. Chapter 2, exercise 4.1.
4. Chapter 2, exercise 4.3.
5. Chapter 2, exercise 5.1.
6. Chapter 2, exercise 5.5.