Canonical metrics and stability of Fano varieties

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Abstract

Finding canonical metrics on compact Kähler varieties has been an intense topic of research for decades. A famous result of Yau says that every compact Kähler manifold with non-positive first Chern class admits a Kähler-Einstein metric (when the Chern class is negative this was also independently proved by Aubin). In this talk, I’ll present some recent joint works with Hamid Abban, Yuchen Liu and Chenyang Xu on the existence of Kähler-Einstein metrics when the first Chern class is positive and the variety is possibly singular (such varieties are called Fano varieties). I’ll focus on two particular aspects: the solution of the Yau-Tian-Donaldson conjecture, which predicts that the existence of Kähler-Einstein metrics on Fano varieties is equivalent to an algebro-geometric stability condition called K-polystability, and a systematic approach (using birational geometry) to decide whether Kähler-Einstein metrics exist on explicit Fano varieties.