Homework due Friday, January 28, at 11:00 pm Pacific Time.

A. Let $f$ be a bounded function on $[a, b]$. Assume that $f \in \mathcal{R}$ on $[a, b]$.

1. Prove that $f$ is continuous at some point $x \in [a, b]$.
2. Prove that $f$ is continuous on a dense subset of $[a, b]$.

(Hint: For part (1) start by showing that there exists a subinterval $I_1 = [a_1, b_1] \subset [a, b]$ so that $\sup I_1 f - \inf I_1 f \leq 1$; note that we may replace $[a_1, b_1]$ by a smaller subinterval and assume $I_1 = (a_1, b_1)$. Repeat this process and construct a sequence $I_1 = [a_1, b_1] \supset I_2 = [a_2, b_2] \supset \cdots$ of closed intervals so that $[a_{n+1}, b_{n+1}] \subset (a_n, b_n)$ and $\sup I_n f - \inf I_n f \leq 1/2^{n-1}$ for all $n$. Show that $f$ is continuous at $\cap_{n=1}^{\infty} I_n$.

For part (2) use part (1) for arbitrary subintervals of $[a, b]$.)

B. Rudin, Chapter 6 (page 147), problems # 3, 8, 10, 11.

The following problems are for your practice, and will not be graded.

1. Rudin, Chapter 6 (page 147), problem #6.