Review 2

Please do not make any assumptions about the composition of the final exam from this set of review problems. Do not assume that the exam questions will be exactly as the questions below, or slight modifications of them. The test problems may look completely different, but if you are able to solve the review problems (closed book, closed notes) then you have necessary knowledge and skills to do well on the final. Also this set is not an indication of how many problems of each type you will encounter on the exam.

You can find the solutions outside my office (KH 012) starting tomorrow morning at 11am.

Number	Points	Score
1	20	
2	50	
3	10	
4	30	
5	40	
6	40	
7	30	
8	30	
9	40	
Total	300	

Good luck! ☺

1. (20 points) Solve the system:

$$\begin{aligned} 3x + 2y - z &= 5 \\ 2x + y + z &= 4 \end{aligned}$$

2. (50 points) Let $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{bmatrix}$.

   (a) Find the eigenvalues and the corresponding eigenspaces of A.

   (b) Find an eigenbasis for A.

   (c) Are the vectors that you found in the previous part orthogonal?

   (d) Decide whether A is diagonalizable or not. If it is, find a matrix S and a diagonal matrix S for which $A = SDS^{-1}$. (Check your answer!) If it is not, justify!

   (e) Find a formula for $A^m \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, where m is any positive integer.

3. (10 points) Find all the real numbers a for which the matrix

$$\begin{bmatrix} 2 & a & 0 \\ -1 & -2 & 4 \\ 2 & 0 & 1 \end{bmatrix}$$

   has 1 as an eigenvalue.

4. (30 points) Let V denote the plane in $\mathbb{R}^3$ spanned by the orthonormal vectors $\vec{u}_1$ and $\vec{u}_2$. Let $T : V \to V$ be the projection onto the line spanned by $\vec{u}_2$. Find the matrix of T with respect to the basis $\mathcal{B} = \{\vec{u}_1, \vec{u}_2\}$.

5. (40 points)

   (a) Find a 3×3 skew-symmetric matrix with determinant 1.

   (b) Check if $Y = \{p(t) \in P_2; p'(0) = 0\}$ is a subspace of P_2.

(c) Check if $L : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$, $L(A) = AA^T - A^T A$, is a linear transformation.

(d) Check if the 6×6 matrices that commute with

$$A = \begin{bmatrix} 2 & 1 & 42 & 0 & 5 & -1 \\ -1 & -2 & 1 & 3 & 34 & \pi \\ 1 & 1 & 1 & 8 & 0 & 0 \\ 13 & -1 & 17 & 85 & 6 & 0 \\ 0 & 0 & 0 & -8 & 1 & 1 \\ 3 & 178 & 1642 & 1848 & 2006 & 5 \end{bmatrix}$$

form a linear subspace of $\mathbb{R}^{6\times 6}$.

6. (40 points) Let $L : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$, $L(A) = 2A + A^T$.

(a) Find the matrix of L with respect to the basis

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \right\}.$$

(b) Find the change of basis matrix from $\mathcal{B}_1$ to

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \right\}.$$

(c) Find the matrix of L with respect to $\mathcal{B}_2$.

(d) Find an eigenbasis for L, if it exists. *(Hint: your work in the previous parts of the problem might be useful)*

7. (30 points) Find a formula for the orthogonal projection on $W^\perp$, where

$$W = Span\left(\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right).$$

8. (30 points) Find the matrix of the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by the rotation about the z-axis through an angle of $\pi/2$ counterclockwise as viewed from the positive z-axis.

9. (40 points)

(a) Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 9 \\ 0 & 0 & 3 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

(b) Compute $\det(3A)$, $\det(-A)$, $\det B$, $\det C$, $\det(A^T)$, $\det AB^{-5}A^TC$, $\det(A+B)$, $\det(A-B)$, where

$$B = \begin{bmatrix} 3 & 2 & 3 & 4 \\ 6 & 4 & 7 & 9 \\ 0 & 0 & 3 & 0 \\ 3 & 1 & 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 3 & 6 & 10 & 13 \\ 2 & 4 & 7 & 9 \\ 0 & 0 & 3 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Hint: how are B and C related to A?