(48)

306

(49) (a) $\det(A-\lambda I_2) = \det\begin{bmatrix} 2-\lambda & 0 \\ 3 & 4-\lambda \end{bmatrix} = (2-\lambda)(4-\lambda)$

So $\lambda = 2$ or $\lambda = 4$

$\underline{\lambda = 2}$ $\quad A - 2I_2 = \begin{bmatrix} 0 & 0 \\ 3 & 2 \end{bmatrix}$

$\text{rref}(A-2I_2) = \begin{bmatrix} 1 & 2/3 \\ 0 & 0 \end{bmatrix}$

$\text{Im}(A-2I_2) = \text{Span}\left(\begin{bmatrix} 0 \\ 3 \end{bmatrix}\right) = y\text{-axis}$

$\ker(A-2I_2) =$ line spanned by $\begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix}$ = line of equation $3x+2y=0$

$\underline{\lambda = 4}$ $\quad A - 4I_2 = \begin{bmatrix} -2 & 0 \\ 3 & 0 \end{bmatrix}$

$\text{rref}(A-4I_2) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$\text{Im}(A-4I_2) = \text{Span}\left(\begin{bmatrix} -2 \\ 3 \end{bmatrix}\right) =$ line spanned by $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

= line of equation $3x+2y=0$

$\ker(A-4I_2) =$ line spanned by $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ = y-axis

(b) $\det(A-\lambda I_2) = \det\begin{bmatrix} -6-\lambda & 6 \\ -15 & 13-\lambda \end{bmatrix} = 12 - 7\lambda + \lambda^2$

So $\lambda = 3$ or $\lambda = 4$

$\underline{\lambda = 3}$ $\quad A - 3I_2 = \begin{bmatrix} -9 & 6 \\ -15 & 10 \end{bmatrix}$

$\text{rref}(A-3I_2) = \begin{bmatrix} 1 & -2/3 \\ 0 & 0 \end{bmatrix}$

$\text{Im}(A-3I_2) = \text{Span}\left(\begin{bmatrix} -9 \\ -15 \end{bmatrix}\right) =$ line spanned by $\begin{bmatrix} -9 \\ -15 \end{bmatrix}$ = line of equation $5x-3y=0$

$\ker(A-3I_2)$ = line spanned by $\begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$ = line of equation $3x-2y=0$

$\underline{\lambda=4}$ $\quad A-4I_2 = \begin{bmatrix} -10 & 6 \\ -15 & 9 \end{bmatrix}$

$\text{rref}(A-4I_2) = \begin{bmatrix} 1 & -3/5 \\ 0 & 0 \end{bmatrix}$

$\text{Im}(A-4I_2) = \text{Span}\left(\begin{bmatrix} -10 \\ -15 \end{bmatrix}\right)$ = line of equation $3x-2y=0$

$\ker(A-4I_2)$ = line spanned by $\begin{bmatrix} 3/5 \\ 1 \end{bmatrix}$ = line of equation $5x-3y=0$

(50)

(a) $\det(A-\lambda I_3) = 1-\lambda^3$

So $\lambda=1$ or λ some complex number, which we'll ignore

$A-1\cdot I_3 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$

$\text{rref}(A-1\cdot I_3) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$\text{Im}(A-I_3) = \text{Span}\left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}\right)$ = Plane spanned by $\left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}\right)$

= plane of equation $x+y+z=0$

$\ker(A-I_3)$ = line spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $\det(A-\lambda I_3) = 1+\lambda-\lambda^2-\lambda^3 = (1+\lambda)(1-\lambda^2) = (1+\lambda)^2(1-\lambda)$

So $\lambda=1$ or $\lambda=-1$

$\underline{\lambda=1}$ $\quad A-1\cdot I_3 = \begin{bmatrix} -4 & 0 & 4 \\ 0 & -2 & 0 \\ -2 & 7 & 2 \end{bmatrix}$

$$\operatorname{rref}(A - I_3) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\operatorname{Im}(A - I_3) = \operatorname{span}\left(\begin{bmatrix} -4 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 7 \end{bmatrix}\right) = \text{plane of equation } -4x + 28y + 8z = 0$$

$$\text{or } -x + 7y + 2z = 0$$

$$\ker(A - I_3) = \text{line spanned by } \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\underline{\lambda = -1}$

$$A - (-1)I_3 = A + I_3 = \begin{bmatrix} -2 & 0 & 4 \\ 0 & 0 & 0 \\ -2 & 7 & 4 \end{bmatrix}$$

$$\operatorname{rref}(A + I_3) = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\operatorname{Im}(A + I_3) = \operatorname{span}\left(\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}\right) = \text{Plane of equation } y = 0$$

$$= (x, z)\text{-plane}$$

$$\ker(A + I_3) = \text{line spanned by } \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

■