

Review

Please do not make any assumptions about the composition of the exam from this set of review problems. Do not assume that the exam questions will be exactly as the questions below, or slight modifications of them. The test problems may look completely different, but if you are able to solve the review problems (closed book, closed notes) then you have necessary knowledge and skills to do well on the midterm.

Also this set is not an indication of how many problems of each type you will encounter on the exam.

1. Determine whether the indefinite integral

$$\int_0^{\infty} x e^{-2x} dx$$

is convergent or divergent. If it is convergent, evaluate it.

You will probably need to use L'Hospital's rule to compute one of the limits.

2. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2 \cdot 3^n - 3 \cdot 5^n}{2^{3n}}$$

converges or not. If it is convergent, calculate its sum.

3. Is the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$$

convergent? Is it absolutely convergent?

4. Use the comparison test to determine whether the series

$$\sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n)^n}$$

converges or diverges.

5. Find the radius and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-8)^n (x-2)^{3n}}{\sqrt{2n+1}}.$$

6. The power series representation of $\sin x$ is

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot x^{2n+1}$$

for all real x .

Find the power series representation of the function

$$F(x) = \int_0^x \sin(t^2) dt$$

using termwise integration.

Good luck on the midterm! ☺