REVIEW

Please do not make any assumptions about the composition of the midterm from this set of review problems. Do not assume that the exam questions will be exactly as the questions below, or slight modifications of them. The test problems may look completely different, but if you are able to solve the review problems (closed book, closed notes) then you have necessary knowledge and skills to do well on the midterm.

Also this set is not an indication of how many problems of each type you will encounter on the exam.

1. **Improper Integrals**

1. (a) Find the value of the following improper integral:

$$\int_0^\infty \frac{1}{1+x^2}\,dx$$

(b) Use this to show that $\displaystyle\int_0^\infty \frac{\sin^2 x}{1+x^2}\,dx$ converges.

2. **L'Hospital's rule**

2. Compute the following limits

$$\lim_{x\to\infty}\frac{\ln x}{x^{0.01}}, \qquad \lim_{x\to 0}\sqrt{x}\ln x, \qquad \lim_{x\to 0}x^x, \qquad \lim_{x\to\infty}(1+1/x)^{3x}.$$

3. **Series**

3. Which of these series are convergent? Explain.

$$\sum_{n=1}^\infty \frac{\sqrt{n}}{n^2+1}, \qquad \sum_{n=1}^\infty \frac{n!}{2^n}, \qquad \sum_{n=2}^\infty \frac{n}{(\ln n)^n}, \qquad \sum_{n=1}^\infty \frac{n!}{(2n)!}, \qquad \sum_{n=2}^\infty \frac{1}{n(\ln n)^2}.$$

4. Test the following series for convergence.

$$\sum_{n=1}^\infty (-1)^n\cdot\frac{\ln n}{n}, \qquad \sum_{n=1}^\infty (-1)^n\cdot\frac{n}{n^2+2}, \qquad \sum_{n=2}^\infty (-1)^n\cdot\frac{1}{(\ln n)^2}.$$

What can you say about absolute convergence in each case?

5. Find the interval of convergence for the following power series.

$$\sum_{n=1}^\infty \frac{n\,x^{2n}}{3^n}, \qquad \sum_{n=1}^\infty \frac{x^{3n}}{n^2\cdot 64^n}, \qquad \sum_{n=2}^\infty \frac{x^n}{\ln n}, \qquad \sum_{n=2}^\infty \frac{x^n}{n\ln^2 n}.$$

6. Find the radius of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} \cdot x^n.$$

7. Determine whether the following series converge or diverge. You must show **all** your work and state which test you are using.

$$\sum_{n=0}^{\infty} \frac{5^n}{3^n + 4^n}, \qquad \sum_{n=0}^{\infty} \frac{5^n + 3^n}{7^n + 4^n}.$$

Remember, $a^n + b^n \neq (a+b)^n$!

8. Find the sum

$$\sum_{n=3}^{\infty} \frac{3^n + 4^n}{5^n}$$

9. Prove that the series

$$\sum_{n=1}^{\infty} \frac{10^n}{n!}$$

converges. What can you conclude from here about $\lim_{n\to\infty} \frac{10^n}{n!}$?

10. Starting with

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

find the power series representation for $\frac{1}{1+x^2}$. Integrating this series find the power series for $\arctan x$.

Without any calculations, can you find the radius of convergence for the power series for $\arctan x$?

11. Starting with

$$\sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdots (2n-1)}{2^n \cdot n!} \cdot x^n = \frac{1}{\sqrt{1-x}}$$

find the power series representation for $\frac{1}{\sqrt{1-x^2}}$. Integrating this series find the power series for $\arcsin x$.