MATH 17 REVIEW SOLUTIONS, FALL 2004

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1. Differential Equations

1.1. First order equations. Solve for the General solution and then the solution that fits the given initial data, if given.

$$(1+x^2)\frac{dy}{dx} = y^2,\ y(0) = 1$$

Answer: $y = \frac{1}{-\tan^{-}1(x)+1}$.

$$(1-x^2)\frac{dy}{dx} = 2e^y$$

Answer: $y = \ln(\ln(\frac{|x-1|}{|x+1|} + C))$

$$y' + xy = x^3,\ y(1) = 1/6$$

Answer: $y = \frac{x^2}{2} - 1 + Ce^{-x^2/2}$, $C = \frac{2}{3}\sqrt{e}$

$$xy' = y - x$$

Answer:$y = x\ln x + Cx$

1.2. Second order equations. Find general solutions and specific solutions, if appropriate:

$$y'' + 3y' + 2y = -7\sin(2x) + 6\cos(2x),\ y(0) = 0, y'(0) = 0$$

Answer:$y = C_1e^{-2x} + C_2e^{-x} + \frac{27}{20}\cos 2x + \frac{11}{20}\sin 2x$ (But you still have to plug in initial data to solve for C_1 and C_2)

$$y'' + 2y' + y = 0$$

Answer: $y = C_1e^{-x} + C_2xe^{-x}$.

2. Sequences and Series

Recall the difference between a sequence and a series. For sequences remind yourself about the squeeze lemma and L'Hospital's rule. For Series Remind yourself about the n-th term test, geometric series, p-series, the integral test, alternating series test, ratio test, and comparison tests. Also review the difference between absolute convergence and conditional convergence.

2.1. Sequences. Find the following limits:

$\lim_{n\to\infty}\frac{n^{100}}{n^{99}+n^{47}} = \infty$	$\lim_{n\to\infty}\frac{\sin(n)+n}{n+1} = 1$ (think squeeze theorem)
$\lim_{n\to\infty} n^{1/n} = 1$	$\lim_{n\to\infty} n\ln\frac{1}{n} = -\infty$
$\lim_{n\to\infty}\frac{e^n}{n^2} = \infty$	$\lim_{n\to\infty}\frac{\sqrt{n}}{1+\sqrt{n}} = 1$ (multiply by $\sqrt{n}/\sqrt{n}$)

2.2. **Series.** . Determine whether the following series converge or diverge. For geometric series give the sum.

$\sum_{n=0}^{\infty} ne^{-n}$ cvgs $\quad \sum_{n=0}^{\infty} \frac{2^{2n}+3^{3n}}{4^{4n}}$ cvgs $\quad \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$ cvgs

$\sum_{n=0}^{\infty} \frac{1}{n}$ dvgs $\quad \sum_{n=1}^{\infty} \frac{n}{\ln n}$ dvgs (nth term) $\quad \sum_{n=0}^{\infty} \frac{e^n}{3^n-n}$ cvgs

$\sum_{n=0}^{\infty}(-1)^n \tan^{-1} n$ dvgs (nth term) $\quad \sum_{n=0}^{\infty} \frac{(-2)^n}{\pi^n+4}$ cvgs $\quad \sum_{n=0}^{\infty} \frac{n^2}{n!}$ cvgs

3. Power Series

3.1. **Domains of convergence.** Answers.

$\sum_{n=0}^{\infty} \frac{x^n}{n}$ $(-1 \le x < 1)$ $\quad \sum_{n=0}^{\infty} \frac{(x-2)^n}{2^{3n}}$ $(-6 < x < 10)$ $\quad \sum_{n=0}^{\infty} x^{3n}$ $(-1 < x < 1)$

$\sum_{n=0}^{\infty} \frac{(2n)!x^n}{n!}$ $(x = 0)$ $\quad \sum_{n=0}^{\infty} nx^n$ $(-1 < x < 1)$ $\quad \sum_{n=0}^{\infty} \frac{(x+1)^n}{4^n}$ $(-3 < x < 4)$

3.2. **Representing functions as power series.** Answers.
$\int \tan^{-1} x\, dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

3.3. **Taylor series.**

$$f(x) = \frac{1}{\sqrt{x}},\ a = 16$$

Answer: $\frac{1}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n(1\cdot 3\cdot 5 \ldots 2n-1)4^{2n-1}(x-16)^n}{n!}$

$$\sin(2x),\ a = 0$$

Answer: $\sum_{n=0}^{\infty} \frac{2(-4)^n x^{2n+1}}{(2n+1)!}$

3.4. **Potluck.**

$$\int e^{x^2}\, dx = C + \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)n!}$$

$$(x^2+1)\sin x = x + \sum_{n=0}^{\infty}(-1)^n \left(\frac{1}{2n+1} - \frac{1}{2n+3}\right) x^{2n+3}$$

4. Parametric curves

The Curve $y = \cos(2t)$, $x = \sin^2(t)$, $-\infty < t < \infty$ lies inside the line $y = 1 - 2x$, $0 \le x \le 1$. Remember $\sin^2 t = \frac{1}{2} - \frac{1}{2}\cos 2t$.

5. Polar co-ordinates

$r = \theta$ looks like a spiral.
$r = \sin 5\theta$ is a five petaled rose with one petal pointed in the direction $\theta = \pi$
$r = \cos(\theta/2)$ should look almost like an 8 inside an ellipse.