MATH 17 REVIEW, FALL 2004

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1. Differential Equations

The important things to remember from differential equations is how to solve for solutions of first order linear, first order separable, and second order linear equations with constant coefficients. You should be able to both solve initial value problems and find general solutions. You should also review population models.

1.1. **First order equations.** Solve for the General solution and then the solution that fits the given initial data, if given.

$$(1+x^2)\frac{dy}{dx} = y^2,\ y(0) = 1$$
$$(1-x^2)\frac{dy}{dx} = 2e^y$$
$$y' + xy = x^3,\ y(1) = 1/6$$
$$xy' = y - x$$

1.2. **Radioactive decay.** If a radioactive isotope has a half-life of 100 years, how long will it take for 9/10 of a one kilogram slab of material made from the isotope to decay?

1.3. **Population models.** Assume a lake has a carrying capacity of 10,000 fish. If the lake is stocked with 50 fish, and the initial growth rate is 100 fish per year, about how many fish will be in the lake in four years? Assume the logistic equation applies. There should be several messy logs in your answer.

1.4. **Orthogonal trajectories.** Find the orthogonal trajectories to the family of curves $y = \frac{k}{1+x^2}$.

1.5. **Second order equations.** Remind yourself of the three cases for finding the general solution to a second order, homogeneous, linear differential equation with constant coefficients. Recall how the general and particular solution are combined when the right hand side of your equation is not 0.

Find general solutions and specific solutions, if appropriate:

$$y'' + 3y' + 2y = -7\sin(2x) + 6\cos(2x),\ y(0) = 0, y'(0) = 0$$
$$y'' + 4y' + 13y = 10e^{-x}$$
$$y'' + 2y' + y = 0$$

2. Sequences and Series

Recall the difference between a sequence and a series. For sequences remind yourself about the squeeze lemma and L'Hospital's rule. For series, remind yourself about the n-th term test, geometric series, p-series, the integral test, alternating series test, ratio test, and comparison tests. Also review the difference between absolute convergence and conditional convergence.

2.1. **Sequences.** Find the following limits:

$$\lim_{n\to\infty} \frac{n^{100}}{n^{99}+n^{47}} \quad \lim_{n\to\infty} \frac{\sin(n)+n}{n+1}$$
$$\lim_{n\to\infty} n^{1/n} \quad \lim_{n\to\infty} n\ln\frac{1}{n}$$
$$\lim_{n\to\infty} \frac{e^n}{n^2} \quad \lim_{n\to\infty} \frac{\sqrt{n}}{1+\sqrt{n}}$$

2.2. **Series.** Determine whether the following series converge or diverge. For geometric series give the sum.

$$\sum_{n=0}^{\infty} ne^{-n} \quad \sum_{n=0}^{\infty} \frac{2^{2n}+3^{3n}}{4^{4n}} \quad \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$$
$$\sum_{n=0}^{\infty} \frac{1}{n} \quad \sum_{n=2}^{\infty} \frac{n}{\ln n} \quad \sum_{n=0}^{\infty} \frac{e^n}{3^n-n}$$
$$\sum_{n=0}^{\infty} (-1)^n \tan^{-1} n \quad \sum_{n=0}^{\infty} \frac{(-2)^n}{\pi^n+4} \quad \sum_{n=0}^{\infty} \frac{n^2}{n!}$$

You should also try to do the very last problem with a comparison test. (Another method is easier, but you should also be able to do the last problem if you are told to use a comparison test.)

3. Power series

There are two things you should know about power series. The first is how to determine for what x-values a series converges. The other is being able to find power series representations of functions, possibly using Taylor series. You should learn the power series representations of e^x, $\sin x$, $\cos x$, and $1/(1-x)$.

3.1. **Domains of convergence.** For what values of x do the following series converge? Check endpoints.

$\sum_{n=0}^{\infty} \frac{x^n}{n}$ $\sum_{n=0}^{\infty} \frac{(x-2)^n}{2^{3n}}$ $\sum_{n=0}^{\infty} x^{3n}$
$\sum_{n=0}^{\infty} \frac{(2n)!x^n}{n!}$ $\sum_{n=0}^{\infty} nx^n$ $\sum_{n=0}^{\infty} \frac{(x+1)^n}{4^n}$

3.2. **Representing functions as power series.** Find a power series representation of each function and give a center and radius of convergence.
$f(x) = \frac{1}{x^2}$ $f(x) = \ln(e + x)$ $f(x) = \tan^{-1} x$
$f(x) = (3x + 2)^{-2}$ $\int \tan^{-1}(x)\, dx$ $f(x) = x^2 + 2$

3.3. **Taylor series.** Find Taylor series for the following functions with the given center of convergence (a).

$$f(x) = cos(x)\,, a = \pi/4$$
$$f(x) = \ln ex\,, a = 1$$
$$f(x) = \frac{1}{\sqrt{x}}\,, a = 16$$
$$f(x) = \sin 2x\,, a = 0$$

3.4. **Potluck.** Mix and match methods as you see fit to get power series for the following functions. Give centers and radii of convergence.

$$\int e^{x^2}\, dx$$
$$\ln(x^e)$$
$$f(x) = 2^x$$
$$\int \sqrt{1+x}\, dx$$
$$f(x) = (x^2 + 1)\sin(x)$$

4. Parametric Curves

Graph the following curves and find where the tangent line is flat or vertical.

$$y = \cos(2t),\ x = \sin^2(t),\ -\infty < t < \infty$$

$$y = t^3 - t,\ x = t^2$$

For this second curve, determine when the graph is concave up.

Show that the unit circle has perimeter 2π by finding the length of

$$y = \sin t,\ x = \cos t,\ 0 \le t \le 2\pi.$$

5. Polar Coordinates

You should be comfortable graphing functions in polar coordinates and finding areas in polar coordinates.

5.1. **Graphing.** Graph the following functions:

$$r = \theta,\ 0 \le \theta \le \pi$$

$$r = \sin 5\theta$$

$$r = \cos(\theta/2)$$

5.2. **Areas.** Find the area in one loop of $r = \sin 6\theta$.

Find the area outside the circle $r = 1$ and inside the circle $r = 2\cos\theta$. The answer is $\frac{\pi}{6} + \frac{\sqrt{3}}{2}$.